# A METHODOLOGY UTILIZING SEMANTIC INFORMATION MEASURES FOR CONVERSATIONAL OR DIALOGUE EXPERIMENTS

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# A METHODOLOGY UTILIZING SEMANTIC INFORMATION MEASURES FOR DIALOGUE EXPERIMENTS

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## SUMMARY

This dissertation provides a linguistically oriented methodology for conducting experiments in which the subject may play an active role in a dialogue or conversation on a trial by trial basis. A distinction is made between two levels of analysis. A macro-analysis which utilizes semantic information measures (SIM) to assess the status of information flow between the subject and an information source, and a micro-analysis which provides a processor program description of individual subject strategies in acquisition and retention.

A model of the interaction of short term and long term memory is presented which is based on a micro-level analysis of obtained SIM data. An analysis of observed subject strategies and theoretically optimal strategies is presented. The methodology is demonstrated by application to experimental data.

# CHAPTER I

# THE METHODOLOGY

In traditional experiments requiring information acquisition (such as learning, concept formation, etc.) trials are presented in a fixed or random order. This procedure allows the subject (S) little or no opportunity to control the sequence of cummunication events and thereby restricts the range of behavior displayed. For example, such procedures limit the subject's information acquisition strategies. As a result, conclusions drawn from traditional experiments may seriously underestimate the importance of subject directed activities in acquiring information and present a distorted picture of human information processing. This paper provides a linguistically-oriented methodology for conducting experiments in which the subject may play an active role in a dialogue or conversation on a trial by trial basis. This methodology involves the utilization of semantic information measures (SIM) to assess the status of information flow between the subject and an information source as defined by the experimental situation.

## Experimental Background

The problem of assessing and facilitating S's information acquisition is emphasized by recent research on man-machine interaction. This is illustrated by the work of Gordon Pask (1968) on adaptive machines which indicates substantial differences in human information acquisition strategies. Dexter [1972] has also investigated the variability of strategies in the context of computer-aided instruction. She finds substantial differences not only in strategies, but also in their effectiveness in learning tasks. This conclusion points to the need for a methodology which permits the design of experimental situations in which there is a clear display of the S's information acquisition strategy. Given an adequate display of S's strategy, the design of an appropriate adaptive system becomes feasible.

Semantic Information Measures And Communication Events

In 1950, Rudolf Carnap [1950] published Logical Foundations of Probability in which he demonstrated an important technique for assigning probabilities as degrees of confirmation to sentences in a formal language. This interpretation of probabilities is known as logical probability and must, according to Carnap, be understood as distinct from probability interpreted as a relative frequency measure. This development in inductive logic permitted Carnap and Bar Hillel [1964] to outline a semantic information measure utilizing logical probabilities in a manner similar to information measures developed by Shannon and Weaver [1949]. However, it was emphatically pointed out by Bar-Hillel that the information measures of Shannon and Weaver could not be considered semantic measures of information; e.g. Bar-Hillel [1964], pp. 283-290.

The work by Carnap and Bar-Hillel was limited to lower order, unquantified languages. Application of the measures developed by Carnap and Bar-Hillel, therefore, only apply to universal sentences due to this restriction.

Recently David Harrah [1963a] [1963b] developed a logical model of communication which is of interest in information acquisition situations. This model explicates questions and answers in a formal language which permit the calculation of semantic information measures.

The basic assumption about measures of information is that information is the reduction of uncertainty. In order to measure uncertainty, distinctions are made between the logical possibilities that can be expressed within a language $\mathcal{L}$. The more possibilities a sentence (Sen) of $\mathcal{L}$ admits the more probable it is in the logical sense of probability. The more possibilities a Sen of $\mathcal{L}$ excludes, the less uncertainty it leaves and hence the more information it conveys. Thus, information is a monotonically decreasing function of logical probability. In addition, it is reasonable to require that information should be additive for sentences that are logically independent. It is also assumed that information be non-negative, that it should be zero for logical theorems of $\mathcal{L}$ and be constant over any class of logically equivalent sentences of $\mathcal{L}$.

Suppose that language $\mathcal{L}$ is a lower predicate calculus with the usual connectives, variables and quantifiers. Further suppose that it has m monadic logically independent primitive predicates $P_1, P_2, \ldots, P_m$; and that it has n primitive individual constants $a_1, a_2, \ldots, a_n$, that name n individuals.

In such a language, it is possible to give a function P which attaches to each Sen a logical probability P(Sen). This function is presented below.

Given the requirements outlined above for an information measure and the probability function P, at least two uncertainty functions are available. These are:

1) Content: Cont(Sen) = 1-P(Sen)

2) Information: Inf(Sen) = -log(P(Sen)).

(The base of the logarithm is generally taken as 2.)

These two measures satisfy the basic requirements of an information measure and both seem to have potential merit. In comparison: (1) the content function is a very direct measure of the number of alternatives that Sen excludes, while (2) the information function is the same as the function based on the frequency probability interpretation and used by Shannon-Weaver in statistical information theory. This Inf function gives rise to the familiar entropy equation

$$H = \sum_i p_i \log P_i$$

for expected information in situations where a number of mutually exclusive alternatives with probabilities $P_i (i=1,2,\ldots)$ present themselves.

These measures are related as follows:

$$\text{Inf(Sen)} = \log \frac{1}{1 - \text{Cont(Sen)}}$$

Bar-Hillel [1964] has said that Cont(Sen) is a measure of the substantive information conveyed by Sen while Inf(Sen) is considered the surprise of value of Sen.

These measures give rise to additional measures such as incremental information, transmitted information and conditional information as will be discussed later.

In many experimental situations, the language between the subject (S) and the experimenter (E) may be formalized as a lower predicate calculus. For example, one class of experiments may require S to map a set of stimuli ($s_1 \ldots s_m$) onto a set of response items ($r_1 \ldots r_n$) according to a predetermined rule. For example, in such a case the language of the experiment might be formalized as the standard lower predicate calculus with M independent, monadic predicates $P_1, P_2 \ldots P_m$ and N individuals $a_1, a_2, \ldots, a_n$. Such a language is generally referred to as an $\mathcal{L}_n^m$ language. The interpretation of $\mathcal{L}_n^m$ is that the predicates $P_1, \ldots, P_m$ are taken to represent stimulus components of the matching rule which match stimuli $S_1, \ldots, S_m$ to responses $r_1, \ldots, r_n$. The individuals $a_1, \ldots, a_n$ represent the responses $r_1, \ldots, r_n$. Accordingly, $P_3a_2$ would be interpreted as the statement "stimulus $S_3$ is matched to response $r_2$."

In such a language quantification can be eliminated in favor of enumeration. For example, $(\exists x)$ (Px) where Px is any compound formula formed from atomic formulas $P_1x, \ldots, P_mx$ and the usual logical connectives can be written:

$$Pa_1 \vee Pa_2 \vee \ldots \vee Pa_n$$

(x) (Px) can be written:

$$Pa_1 \wedge Pa_2 \wedge \ldots \wedge Pa_n$$

An effect of the quantification by enumeration is to permit the calculation of inductive probabilities by the methods of Carnap rather than those of Hintikka.[1]

The method of calculating P(Sen) for sentences of $\mathcal{L}_n^m$ is as follows:

1. Form $2^m$ Q predicates:

   $Q_i x = \pm P_1 x \wedge \pm P_2 x \wedge \cdots \wedge \pm P_m x$

   $(0 \leq i \leq 2^m)$ For each distribution of - (not) or + (nothing) in front of the Atomic Formulae $P_1x, \ldots, P_mx$

2. Form $2^{m.n}$ Ordinary Constituents or state descriptions:

$$
\begin{aligned}
C_i = & \pm P_1a_1 \wedge \pm P_2a_1 \wedge \cdots \wedge \pm P_ma_1 \\
& \pm P_1a_2 \wedge \pm P_2a_2 \wedge \cdots \wedge \pm P_ma_2 \wedge \cdots \wedge \\
& \quad \vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
& \pm P_1a_n \wedge \pm P_2a_n \wedge \cdots \wedge + P_ma_n
\end{aligned}
$$

   $0 \leq i \leq 2^{m.n}$ For every distribution of 'not' or nothing before the m.n instantiations of the predicates $P_1x, \cdots, P_mx$ with the individuals $a_1, \cdots, a_n$. The reader can easily verify that each $C_i$ is a conjunction of Q predicates instantiated with the individuals. Or:

$$
\begin{aligned}
C = & Q_{i_1}a_1 \wedge \cdots \wedge Q_{i_1}a_n \wedge Q_{i_2}a_1 \wedge \cdots \wedge Q_{i_2}a_n \wedge \cdots \wedge \\
& Q_{i_2m}a_1 \wedge \cdots \wedge Q_{i_2m}a_n
\end{aligned}
$$

[1]In 1963, Jaakko Hintikka [1965] produced a distributive normal form for general sentences which permitted the extensions of semantic information measures to quantified languages.

Thus every sentence Sen of $\mathcal{L}_n^m$ is logically equivalent to a disjunction of ordinary constituents.

Computational Procedure (1)

It is possible to distribute probabilities evenly to the $C_i$ so that for this particular language $P(C_i) = \frac{1}{2^{m.n}}$. Under this scheme to calculate P(Sen) for Sen in $\mathcal{L}_n^m$ we only need find the width of Sen (w(Sen)) where w(Sen) is defined to be the number of ordinary constituents in the disjunction of $C_i$'s logically equivalent to Sen. Then simply

$$P(\text{Sen}) = \frac{w(\text{Sen})}{2^{m.n}}$$

This procedure is used in calculating results given in this paper, and is essentially Carnap's $M^{\dagger}$ proper M function.

Computational Procedure (2)

A second and computationally more difficult procedure is as follows:

1. Let $C_i$ be called isomorphic to $C_j$ if and only if $C_j$ can be obtained from $C_i$ by permutation the individuals $a_1, \ldots, a_n$. Call this isomorphism ISO.
2. Let CONS be the set of all ordinary constituents of $\mathcal{L}_n^m$. Quotient the set CONS by ISO. Then if $x \in$ CONS/ISO, then $\vee x$ (the disjunction of the elements of x) is called a structural description.
3. Let the cardinal of $x \in$ CONS/ISO be $k_x$, then the probability mass of x is evenly distributed among the constituents $C_i \in x$.

Thus:

$$P(C_i) = \frac{1}{2^{m.n}\,k_x}$$

4. Thus for sentences s of ${}^{m}_{n}$

$$P(s) = \sum_{j=1}^{w(s)} P(C_{i_j})$$

This procedure outlined above has the possible advantage that the patterns of assignment of the individuals of the language to the response items do not yield different probabilities (and hence different information content measures) for the sentences of $\mathcal{L}^{m}_{n}$. The quotient of the set CONS by ISO insures that the calculations are not sensitive to these assignments. This procedure is based on Carnap's $M^*$ proper m function; e.g. Carnap [1950]. This $M^*$ function was intended to allow an information user to learn from inductive samples. There are three reasons why the first procedure based on the $M^\dagger$ functions chosen. First, rules are chosen in demonstration tasks in a random fashion and subjects are so informed. Since rules are state descriptions, their random selection might be reflected in equal probability assignments to state descriptions. Second, since S's are informed that they should not expect to discover a predictable rule based on evidence, the $M^*$ function intended for inductive applications would be inappropriate. In fact $M^\dagger$ is a function reserved for deductive applications. It has been referred to as $M_D$ to emphasize its deductive character; e.g. Carnap and Bar-Hillel [1964] and its use is therefore appropriate. Third, empirical studies by Cohen [1970] suggest that the $M^{\dagger}$ function has more psychological validity for the tasks under consideration.

For the calculations represented in the traces in Chapter II the function Inf was chosen as a basic function but a derived function, IncInf was used for the SIM curve. This is:

$$\mathrm{IncInf}(s/h) = \mathrm{Inf}(s \wedge h) - \mathrm{Inf}(h)$$

IncInf(s/h) measures the information of $s \wedge h$ above that of h alone. Here h is the "history" or the conjunction of all previous sentences delivered in a sequence of communication events. A plot of the accumulation of incremental Inf against communication events reveals a trace of information flow to the subject. This shows (1) the points at which information is made available, (2) the amount of information received at each event, (3) the occurrence of events which convey no new information, and (4) the point at which the maximum or total amount of information is available to the subject.

## Communication Events Or Trials

It is important that this procedure permits the calculation of the information content of a question. Intuitively it might appear that questions convey no information, but a question is a communication event in the formal language and can potentially have non-zero information relative to the instructions. Notice that the calculation for traces can be done at this level of resolution. Each communication event can be divided into two communication events which are (1) the question Q and (2) the information resulting from the evaluation of the answer to Q (Inf(Q) if the answer is affirmative and Inf(-Q) if the answer is negative.)

The following logical analysis of questions is based on a suggestion by Harray [1963a] although it is not identical with his procedure.

All forms of questions in our explication are special cases of Harrah's "whether" or disjunctive question, Harrah first describes a prime disjunct as follows:

given a finite sequence $G_1, \ldots, G_n$ of wff $(n > 1)$
for each i $(1 \leq i \leq n)$ write:

$$'G_i^{*}' \text{ for } '((\ldots((((\ldots(-G_1 + -G_2) + \ldots) + -G_{i-1}) +G_i) + -G_{i+1}) + \ldots) + -G_m)'$$

He then enters a definition of a prime disjunction:

(His) Def 7.1

F is a prime disjunction in $G_1, \ldots, G_n$
if and only if $N > 1$ and F is

$$(((\ldots((G_1^{*} \vee G_2^{*}) \vee G_3^{*}) \vee \ldots) \vee G_{n-1}^{*}) \vee G_n^{*})$$

and defines a disjunctive question:

(His) Def 7.2

F is an n place disjunctive question
if and only if F is a prime disjunction in

some sequence $G_1$, ..., $G_n$ of wffs.

and explicates answers to disjunctive questions:

(His) Def 7.3

A direct answer to a disjunctive question, q, is a disjunct of q.

These definitions insure that there is exactly one disjunctive question in each sequence of wff's $G_1$ ..., $G_n$ and that there is at most one true direct answer to it.

Harrah also defines a metalinguistic notation for disjunctive questions.

(His) Def 7.9

(1) a disjunctive question in $G_1$, ..., $G_n$ is referred to as '($G_1$, ..., $G_n$?)'

(2) 'G?' for '(G,-G?)'

This question mark, '?', occurs only metalinguistically and only as an abbreviational device. It is convenient but dispensable, thereby causing Harrah's question logic to differ from question logics of Kubinski [1958], Steinmann [1959] and others.

There are two variations of the same procedure presented; the first is utilized in cases in which S poses questions and the second formalizes questions posed by E.

1. A question by S is taken to be of three types (a) simple, (b) disjunctive, and (c) conjunctive.

A simple question is of the form "does $s_i$ match $r_j$?" and is formalized as $P_i a_j$. It is useful to identify questions with sentences of $\mathcal{L}$ in this manner because it permits an SIM to be calculated on questions as well as on responses.

A disjunctive question is taken to be a question of the form "does $s_i$ match $r_i$ or $r_j$ or ... or $r_k$?" Such questions are formalized as $P_i a_i \vee P_i a_j \vee P_i a_k$. If S were to ask "does $s_i$ or $s_j$ match $r_k$?" then the formalization would be $P_i r_k \vee P_j r_k$.

A conjunctive question is as above using "$\wedge$" for "$\vee$". For example, if S asks "does $s_i$ match $a_j$ and $a_r$?" then the question is taken in $\mathcal{L}^m_n$ to be $P_i a_j \wedge P_i a_r$.

Responses from E which evaluate or answer S's questions are taken to be the same as the question or its negation according as to whether the answer is "yes" or "no." If a question is formalized as q and is answered "yes" then q also is the $\mathcal{L}^m_n$ representation of the response. If the question q is answered "no" then this response is taken to be -q.

Notice that questions that we refer to as conjunctive and questions referred to as disjunctive are both disjunctive questions in Harrah's sense. For example, the question $P_i a_j \wedge P_i a_r$ above is equivalent to Harrah's

$$((P_i a_j \wedge P_i s_r), - (P_i a_j \wedge P_i a_r)?)$$

All questions in our sense are of the type (G,-G?) in Harrah's sense. The responses given to S's questions by E are direct answers to disjunctive questions according to Harrah's def. 7.3.

2. Questions posed by E of the form "what r does $s_i$ match to?" are formalized as $(\exists x)(P_i x)$ but as outlined above the quantification is eliminated in favor of enumeration. This question then becomes:

$$P_i a_1 \vee P_i a_2 \vee \cdots \vee P_i a_n$$

S might respond, for example "$S_i$ matches $r_5$," which is formalized as $P_i a_5$. If S's response is correct then the message from E to S to this effect is formalized also as $P_i a_5$. If S is told by E that he is incorrect then this message is formalized as $-P_i a_5$. Compound questions and responses to such questions can also be formalized.

## Application to Experimental Tasks

In the class of rule learning tasks mentioned above which require S to map a set of m stimuli to a set of n responses, the rules that are possible are exactly the $2^{m.n}$ state descriptions of $\mathcal{L}^m_n$. Thus each possible rule is a statement that each stimulus is or is not mapped on to each response. As a result, with no instructions whatever, except that a mapping rule is to be discovered, S is confronted with a Problem Space (P.S.) which is the disjunction of the $2^{m.n}$ state descriptions or ordinary constituents. Each communication from E to S by way of instructions concerning the nature of the rule or in the form of answers to questions from S about the rule limits the P.S. by eliminating ordinary constituents from this disjunction. The Inf(Sen) function is a tool for evaluating the way S elicits information about this P.S. When S has elicited enough information to fix the rule, he has eliminated all but one ordinary constituent as being inconsistent with this elicited information and therefore has a complete description of his "universe" which is the problem or rule that was embedded in the original P.S.

The use of the $\mathcal{L}_n^m$ formalization of the experimental language provides a means of incorporating a description of the problem space into the language of the experiment and hence allows a measure of the information to S contained in the instructions as they relate to the problem space. This is an important aspect of the methodology. If there is to be an adequate theory of S's search through his problem space, then there must be a means of determining what that problem space is for S. Therefore, it becomes important that the experimental language be capable of containing a description of the problem space and that there be a means of measuring the information that it conveys to S about the task. In the conducted experiments this was included in the instruction to S: "The rule that you are to discover is a one-one rule. This means that one (stimulus item) matches exactly one (response item) and that no two (stimulus items) match the same (response item). There is for each (stimulus) a unique (response)."[2]

The protocols of these experiments indicate that this task description did in fact cause all S's to act upon the same problem

[2] The complete instructions will be found in Appendix B.

space. That is to say that each S understood the nature of the rule he was seeking. This is evidenced by the fact that errors attributable to misunderstanding of the nature of the rule prescribed by the initial instructions were rare. That is to say Ss always queried E about each stimulus item until a pair of matches were discovered and then left that stimulus for another (except for rehearsals of those matches).

Therefore, we must be able to formally describe in $\mathcal{L}_n^m$ the problem space permitted by the natural language excerpt cited above. Toward this end, a description is entered here of the language of the experiments in which the instructions are formalized as an axiom.

## Description of $\mathcal{L}$

1. $\mathcal{L}$ is a family of languages with the following description
   a. The logical frame of all languages in $\mathcal{L}$ is the lower predicate calculus with identity together with the usual model theoretic semantics.
   b. Each $\mathcal{L}_{2n}^R$ member of $\mathcal{L}$ has the following primitive vocabulary (non-logical vocabulary in addition to the vocabulary of its frame)
      i. individual constants: '$a_1$', '$a_2$', ..., '$a_n$', '$b_1$', '$b_2$', ..., '$b_n$'.
      ii. one two place predicator: 'R'
   c. Each $\mathcal{L}_{2n}^R$ member of $\mathcal{L}$ has the following descriptive axiom (in addition to the proof theory of its frame).

      Ax1. $(x)\ (x=a_1 \vee x=a_2 \vee \ldots \vee x=a_n \vee x=b_1 \vee x=b_2 \vee \ldots \vee x=b_n)$

Ax2. $a_1 \neq a_2 \wedge a_1 \neq a_3 \wedge \ldots \wedge a_1 \neq a_n \wedge a_2 \neq a_3 \wedge \ldots \wedge a_2 \neq a_n \wedge \ldots$

$a_{n-1} \neq a_n \wedge b_1 \neq b_2 \wedge b_1 \neq b_3 \wedge \ldots \wedge b_1 \neq b_n \wedge b_2 \neq b_3 \wedge \ldots \wedge b_2 \neq b_n$

$b_{n-1} \neq b_n \wedge a_1 \neq b_1 \wedge a_1 \neq b_2 \wedge \ldots \wedge a_1 \neq b_n \wedge a_2 \neq b_1 \wedge \ldots \wedge a_2 \neq b_n \wedge$

$\ldots a_n \neq b_n$. (an axiom that states that all 2n individuals are distinct)

Ax3. $Ra_1b_1 \wedge Ra_2b_2 \wedge \ldots \wedge Ra_nb_n$ (or alternative)

Zx4. $(x)\,(y)\,(z)\,((Rxy \wedge Rxz \supset y=z) \wedge (Ryx \wedge Rzx \supset y=z))$ one-one of R (or alternative).

d. The following defined notation is introduced:

$\ulcorner P_i x \urcorner$ stands for $\ulcorner Ra_i x \urcorner$ for every i

All the models of $\mathcal{L}^R_{2n}$ have carriers with 2n members and a structure composed of a one-one relation Q which is one-one, whose field is the carrier and whose domain and range are disjoint. For the sake of simplicity all inductive probabilities are calculated by first eliminating quantification by enumeration and then translating all primitive non-logical notation into statements containing only the predicators '$P_i$' and the individual constants '$b_i$'.

2. The language used in the experimental transactions (questions, answers) are formalized in the defined notation of the appropriate $\mathcal{L}^R_{2n}$.
3. The instructions given to the experimental subjects are formalized by the axioms of the appropriate $\mathcal{L}^R_{2n}$ and the inductive probabilities calculated as above.

## Formal Descriptions of the Problem Space

In the description of the language given above a sample axiom, $Ax_4$, for the nature of the rule is given. This sample axiom characterizes the P.S. as the set of all one-one rules. Since the calculation of the information measures is carried out in languages with monadic predicates, we expand here on descriptions of P.S.

With a binary predicate 'R', one-one-ness of the corresponding relation is expressed in the object language by:

$$(x)\ (y)\ (z)\ ((Rxy \wedge Rxz \supset y{=}z) \wedge (Ryx \wedge Rzx \supset y{=}z))$$

Going back to monadic predicates the same fact is metalinguistically expressed by:

$\ulcorner P_x a_y \wedge P_x a_z \urcorner$ entails y=z and $\ulcorner P_y z_x \wedge P_z a_x \urcorner$ entails y=z for all indices x,y,z, where '$P_x$' '$a_y$' are metalinguistic structural descriptions for x=1,2,...,m y=1,2, ..., n respectively of monadic predicates and individual constants of the object language $\mathcal{L}^m_n$. The above metalinguistic entailments are many-one is encased in:

$$\bigwedge_{x}^{m} \bigwedge_{y \neq z}^{n} -(P_x a_y \wedge P_x a_z)$$

one-many is encased in:

$$\bigwedge_{x}^{n} \bigwedge_{y \neq z}^{m} -(P_y a_x \wedge P_z a_x)$$

and that to every element of the domain there is an element of the range and conversely is encased in:

$$\bigwedge_{x}^{m} \bigvee_{y}^{n} P_x a_y \wedge \bigwedge_{x}^{n} \bigvee_{y}^{m} P_y a_x$$

The fact that for every element in the range of R there exists at least and at most two elements of the domain of R related to it is expressed in the object language with the predicate 'R' as follows:

$$(x)\ (\exists y)\ (\exists z)\ (y \neq z \wedge Ryx \wedge Rzx \wedge (w)(Rwx \supset w=y \vee w=z))$$

which, following essentially the same procedures, goes into a metalinguistic schematism as follows:

$$\bigwedge_{x}^{n} \left\{ \bigvee_{y \neq z}^{m} \left[ \bigwedge_{\substack{w \neq z \\ w \neq y}}^{m} P_y a_x \wedge P_z a_x \wedge - P_w a_x \right] \right\}$$

If other instructions are read to S indicating types of rules other than one-one rules, then other axioms must be constructed after the above manner which capture the nature of the alternative rules and substituted for A x 4 above.

An alternate assumption could be used to deal with the information contained in the instruction. The position could be taken that S does not begin the experiment with Inf(P.S.), but rather uses P.S. to skew his probability distribution in such a manner as to begin the experiment with zero information relative to the learning task. Thus S might assign zero probability to the ordinary constituents that are excluded by P.S. (if P.S. $\wedge\ C_i$ is logically false, then $P(C_i) = 0$) and assign equal probabilities to those constituents that are logically compatible with P.S., thus using P.S. to effect a task. However, it turns out that this is not a significantly different procedure, at least for the intended applications. In order to compare this procedure with the one outlined above, consider an $\mathcal{L}^m_n$ language with a one - C rule where $\binom{n}{c} \geq m$. This situation allows a problem space with $w(P.S.) = \binom{n}{c}!/\left(\binom{n}{c} - m\right)$ since the arbitrary first stimulus has $\binom{n}{c}$ possible matches.

Now, if Carnap's $M^\dagger$ function is utilized, each $C_i$ is assigned $M^\dagger\,(C_i) = \frac{1}{2^{m.n}}$ and therefore

$$P(P.S.) = \frac{\binom{n}{c}!/\left(\binom{n}{c}-m\right)}{2^{m.n}}$$

Now let us suppose that a sentence s is processed which eliminates K constituents. The result is that:

$$P(s|P.S.) = \frac{\binom{n}{c}!/\left(\binom{n}{c}-m\right) - K}{2^{m.n}}$$

and

$$\text{IncInf}(s/P.S.) = -\log_2(P(s \wedge P.S.) + \log_2(P(P.S.))$$

$$= m.n - \log_2\left(\binom{n}{c}!/\left(\binom{n}{c}-m\right)\right) - K$$

$$+ \log_2\left(\binom{n}{c}!/\binom{n}{c}-m\right) - m.n$$

$$= \log_2\left(\frac{\dfrac{\binom{n}{c}!}{\binom{n}{c} - M}}{\dfrac{\binom{n}{c}!}{\binom{n}{c} - M} - K}\right) .$$

However, let this new M function, call it $M_{P.S.}$, be used which assigns to each constituent in P.S. equal weight but assigns zero to all others.

Then $P(P.S.) = 1$ and

$$M_{P.S.}(C_i) = \frac{\binom{n}{c} - M}{\binom{n}{c}!}$$

as there are

$$\frac{\binom{n}{c}!}{\binom{n}{c} - M}$$

possible one - C rules in P.S.

Now $P(P.S.) = 1$ and $Inf(P.S.) = 0$ as opposed to $Inf(P.S.) = m \cdot n - \log_2\left(\binom{n}{c}!/\left(\binom{n}{c} - M\right)\right)$ for the $M'$ function. However, we can show that the increments of information are the same. Here when s is processed eliminating K constituents from P.S. we have

$$P(s \wedge P.S.) = \frac{\binom{n}{c}!/\left(\binom{n}{c} - m\right) - K}{\binom{n}{c}!/\left(\binom{n}{c}\right) - M}$$

and $\text{IncInf}(s/P.S.) = -\log_2 (P(s \wedge P.S.)) + \log_2 P(P.S.)$

$$= + \log_2 \frac{\binom{n}{c}!/\left(\binom{n}{c} - M\right)}{\binom{n}{c}!/\left(\binom{n}{c} - M\right) - K} - 0$$

as before.

We see, therefore, that these choices of an M function, the one being Carnap's $M^{\dagger}$ and the other the idiosyncratic $M_{P.S.}$, are both equally usable in a context in which IncInf is of interest. The major difference is that in the one case S starts with Inf(P.S.) and in the other with 0 information and from this new base builds his information state as a sum of increments of information.

One difference is sufficient to cause us to reject this $M_{P.S.}$ in favor of $M^{\dagger}$. That is the fact that $M_{P.S.}$ is not a proper M-function in Carnap's system. One requirement of a proper M-function is that for all state descriptions $C_i$, $M(C_i) > 0$. This is not the case with $M_{P.S.}$ and so we reject it even though it seems otherwise plausible and even appealing.

This methodology allows careful distinction between communications events (QA exchange) that are acquisitions of new information and rehearsals. Any trial i for which $\text{IncInf}(\text{Sen}_i \mid h_i) > 0$ is a trial on which new information has been acquired and therefore is not rehearsal. All other trials are defined to be rehearsal trials. It is not necessarily the case that this distinction reflects the intention of S but it is based on purely logical consideration. For an ideal S, any trial for which $\mathrm{IncInf}(sen_i \mid h_i) > 0$ provides a reduction in the P.S. Hence, he has acquired information relevant to his task. Also for the ideal S, any trial for which $\mathrm{IncInf}\ (sen_i \mid h_i) = 0$ fails to change the P.S. Hence, the trial logically is a rehearsal of already available information.

For the ideal S, these definitions are only prescriptive. However, when an S performs his information acquisition with small deviation from a prescriptive base line, we have increasing confidence that the definitions have a high degree of psychological validity.

Based on the foregoing analysis of questions, the problem space and chosen information measure, we can now enter a series of definitions which lead to a performance record of S in a rule learning task.

Definition 1: A <u>trial</u> $Q_i$? is any S originated $\mathcal{L}^m_n$ formalizable communication event which elicits an $\mathcal{L}^m_n$ formalizable response from E.

Definition 2: A rule is an ordinary constituent $C_i$ of $\mathcal{L}^m_n$.

Definition 3: An E message $M_i$ is an $\mathcal{L}^m_n$ response from E to a trial $Q_i$? from S.

For trial $Q_i$? Then $M_i = Q_i$ or $-Q_i$ according to which of $Q_i$ or $-Q_i$ are compatible with the rule set by E.

Definition 4: A history $h_i$ for trial $Q_i$? is a conjunction of P.S.

with E messages $m_1 \ldots, m_{i-1}$.

$$\begin{cases} h_1 = \text{P.S.} \\ h_{i+1} = h_i \quad m_i \end{cases}$$

Definition 5: $Inc_i = IncInf\ (m_i/h_i)$

$Inc_i$ is simply the increment of information that S receives as a result of E's response to his question on trial i.

Definition 6: A <u>trace</u> for a K trial task is a graph of $Inc_i \times i$ for all $i \leq K$.

Definition 7: S's information available state $IAS_i = \sum_{i=1}^{n} Inc_i$.

The difficulty here is that S's actual information state need not be the same on trial i as $IAS_i$. This is due to forgetting, failure to process inferences, etc. For this reason, a description is needed of the actual information state for trial i, ($AIS_i$). It is not immediately clear what this should be. A conservative definition for $AIS_i$ covers only the case where i is a rehearsal trial on which S is recalling his own information state for (apparently) memorial fixing purposes. In order to do this a definition is given to distinguish between acquisition and rehearsal trials.

Definition 8: An acquisition trial is a trial $t_i$ such that $Inc_i > 0$.

Definition 9: A rehearsal trial is a trial $t_i$ such that $Inc_i = 0$.

Here we have a minor difficulty. On trials, $t_i$, on which S can (logically) infer $M_i$ from $h_i$, but not from $h_{i-1}$, it is not clear that S psychologically treats $Q_i$? as a rehearsal. This may occur when all available responses save one have been eliminated for a particular stimulus. S's often ask if that stimulus is matched to the only available response without appearing to realize that the question logically elicits no new information. S's have, however, commented to E that they were aware that such questions were rehearsals in the logical sense.[3] We take these definitions, therefore, to be sufficiently natural for present purposes.

We can now define bounds on the value of S's actual information state

Definition 10: S's actual information state at trial $Q_i$? is $IAS_i \geq AIS_i \geq Inf(M_i)$ if $M_i = Q_i$ and $t_i$ is a rehearsal trial. $AIS_i$ is undefined otherwise.

Note that if $Inf(M_i) = IAS_i$, then $AIS_i = IAS_i$. This occurs if S rehearses all available information. If $Inc(M_i) < IAS_i$, then S has not rehearsed all available information and we only have a minimum value of $AIS_i$. Here S may be able to rehearse all of the information in $IAS_i$, but not choose to do so; hence, we choose $AIS_i \geq Inf(M_i)$ rather than $AIS_i = Inf(M_i)$.

A task is successfully completed when on trial $Q_i$? $AIS_i$ = M.N. This can only occur when $M_i$ is the rule set by E. As noted earlier, all trials beyond this point are rehearsals.

Until S successfully completes the task by stating the rule and hence terminates his trials, he may err. We must, therefore, identify what constitutes error among the conversational trials. S may ask many questions that may be answered "no" that are not errors

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[3]Bruner, et. al. [1965] refer to this as 'reassuring redundancy' see p. 81 ff.

in any sense. If a trial i is a question $Q_i$? for which S cannot infer the answer from the history $h_i$, then an answer of "no" cannot be counted as error. This is simply a message $M_i = - Q_i$ for which $\mathrm{IncInf}(M_i/h_i) > 0$. Therefore, trial i is not error but information bearing. However, if trial i is a rehearsal (i.e., $\mathrm{Inc}_i = 0$), then an answer of "no" implies that S could logically infer $- Q_i$ from $h_i$ and hence constitutes error. Therefore:

Definition 11: A trial $Q_i$? is an error, iff $\mathrm{Inc}_i = 0$, and $M_i = - Q_i$.

The linguistic analysis presented above allows S to frame his own questions on a trial by trial basis. With these procedures more complicated rules such as one-two rules, one-three rules can also be utilized. However, it should be noted that more complex rules may require more extensive computation and may therefore present practical difficulties.

Finally it should be noted that traditional fixed trial procedures can also be described in terms of the communication analysis. In such cases, the subject's information state can be determined by tests which consist of questions posed by the experimenter which elicit information on all possible s-r connections.[4]

Carnap and Bar-Hillel [1964] have pointed out that Semantic Information Measures (SIM) are measures of the information contained

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[4]The logic of the SIM suggests that S should only be tested on s-r connections which have been previously presented to him. SIM can, of course, be calculated for a test of S's information regardless of the correctness of his answers. The issues raised by the calculation of SIM on incorrect responses are beyond the scope of this discussion.

in language structures for an "ideal" language user. By this they mean a user who has perfect memory is completely logical and is able, with these tools, to completely process all the content of any sentence of the language. It might be assumed that the information traces in Chapter II are representative of the information for an "ideal" subject. The curves do not represent information acquired by the subject, but represent the information that would be acquired by an ideal language user "listening in" on the conversation between the experimenter and S.

To the extent that the S can reliably approximate the performance represented in the SIM curve, one has increasing confidence that the S is processing information in accordance with the assumptions made in calculating the SIM. On the other hand, if S's performance indicates serious discrepancies from that of his SIM acquisition curve, then we might assume that the situation presents an information overload. Thus, the SIM may provide a technique for determining those conditions under which S can or cannot operate as an efficient information processor.

As noted earlier, the SIM curves permits us to divide the subject's performance into two distinct periods. The portion of the experiment represented by the curve before it asymptotes can be called the acquisition period. The period following this can be referred to as the practice period since no new information is presented. The point dividing these periods is the first trial at which the curve achieves maximum value. The terminology is based on logical considerations and is not necessarily descriptive of S's psychological processes.

Dialogue questions during the acquisition portion of the curve which provide no increment of information may indicate practice or rehearsal by S on previously presented information. Since no new information can logically be presented during the practice portion of the experiment as noted, measures or tests of the S's ability to perform the criterion task (as shown by the bars) are required if E is to determine S's information state.

# CHAPTER II

## MACRO-ANALYSIS OF PROTOCOLS

A characteristic of the data obtained in this experimental condition of the conversational methodology is that it is very regular and therefore analyzable on several levels. The highest level of analysis by which these data are to be examined might be called a macro-level of analysis. In this analysis, the nature of a subject's performance as to amount of error, information state, and information trace is in question. In a later chapter, a level referred to as a micro-level of analysis will be found. There, certain theoretical mechanisms will be postulated which might account for some of the characteristics pointed to by the macro analysis.

Samples of the protocols obtained from the participating S's are found in Appendix A. Certain notational abbreviations are systematically utilized and are explained there. The protocols were obtained from seven S's performing in 15 tasks ranging from five items to 18 items. The sessions with the S's were recorded and the recordings were later transcribed and summarized into the protocols as sampled in Appendix A. The protocols are numbered and the S for which the protocol is a performance record is identified by number. The protocols include a record of the increments of information and a sum of increments of information. The sum of increments is used to construct the traces of information available to S as a result of his questions over trials. These are displayed in Figure 1 through Figure 15.

Performances recorded in protocols P-3, P-4, P-5, P-6, P-7a, and P-12 are particularly interesting because of the high level of performance that they represent. An examination of other protocols will indicate that Sxs are not always found to produce data of this regularity.

The striking feature of these protocols is the small amount of error. Error is here used in the sense of Definition 11. Negative responses in acquisition trials are not errors according to this definition. The assumption is that if a trial produces a non-zero increment of information from a negative response from E, then S has not erred, but has received useful information. Protocols P-4, P-5, P-6 and P-12, all represent performances in which S has made only one error. This seems remarkable when one considers that S-6 in P-12 has sorted through 18! possible rules in 121 trials in which 55 of them require him to process non-zero increments of information and has made only one error. Consider P-7a in which S-7 has found the correct rule out of 12! rules in 81 trials with no errors. It appears that these Ss have chosen a strategy of acquisition and rehearsal which is absolutely appropriate to both the task presented to them and to their own memory and processing constraints.

The existence of such error-free performance (EFP) is made to seem more remarkable by comparison with S-1 who failed on two occasions to provide EFP in a task involving only five pair items. Compare also the larger task protocols to P-11 in which S-4 commits six errors and then terminates trials as a result of his inability to process information in such a way as to allow progress further in the task. Coulter (1973) has found that many subjects are able to formulate a strategy for generating questions and processing information in such a way that they can complete tasks of 12 item pairs under several conditions of list order, visual display vs. no display, etc. He has found, however, that few subjects are able to perform error free, but that most Sxs commit many errors even though they eventually complete the task.[5]

Another aspect of the performances of these Ss is the number of trials on which they demonstrate that their actual information state is equal to the information available state. For example, S-3 has demonstrated at six points in P-5 that he has processed and stored all information available to him. If consecutive trials of this sort are included, S-3 demonstrates that $AIS_i = IAS_i$ for $i=4, 9, 12, 13, 20, 24$. That is six trials out of 24. The existence of only one error indicates that the actual information state is likely to be maximum in many other trials also.

Long term retention (24 hours) data were taken for the longer tasks. This retention appears to be quite high, but more control would be required before it could be compared with long term retention data available in other experiments. Coulter (1973) has examined the issue of long term retention in several conditions of conversational tasks.

[5]It appears from Coulter's data that even though appropriate acquisition strategies are chosen, Ss may fail to perform with minimal error due to inadequate rehearsal procedures. As a consequence, a very wide range of performance was observed in number of acquisition trials, total time to reach criterion performance, and number of errors.

Table 1 contains an assembly of macro-level analyses of the various protocols.

Table 1. Macro-Level Analysis of Protocols.

| P | S | NUMBER ITEMS | NUMBER TRIALS | NUMBER ERRORS | NUMBER TRIALS FOR WHICH AIS=IAS | 24 HOUR RETENTION |
|---|---|---|---|---|---|---|
| 1 | 1 | 5 | 15 | 1 | 2 | --- |
| 1a | 1 | 5 | 14 | 1 | 5 | --- |
| 2 | 2 | 5 | 14 | 0 | 5 | --- |
| 3 | 3 | 5 | 13 | 0 | 5 | --- |
| 4 | 3 | 6 | 23 | 1 | 8 | --- |
| 5 | 3 | 7 | 24 | 1 | 8 | --- |
| 6 | 3 | 9 | 48 | 1 | 20 | --- |
| 7 | 3 | 11 | 84 | 2 | 23 | 100% |
| 7a | 7 | 12 | 93 | 0 | 32 | --- |
| 8 | 3 | 14 | 135 | 3 | 19 | 85 5/7% |
| 8a | 1 | 14 | 85 | 5 | 21 | 100% |
| 9 | 5 | 14 | 167 | 18 | 6 | 100% |
| 10 | 2 | 18 | 125 | 14 | 23 | 100% |
| 11 | 4 | 18 | 68 (failed) | 7 | 10 | --- |
| 12 | 6 | 18 | 121 | 1 | 16 | --- |

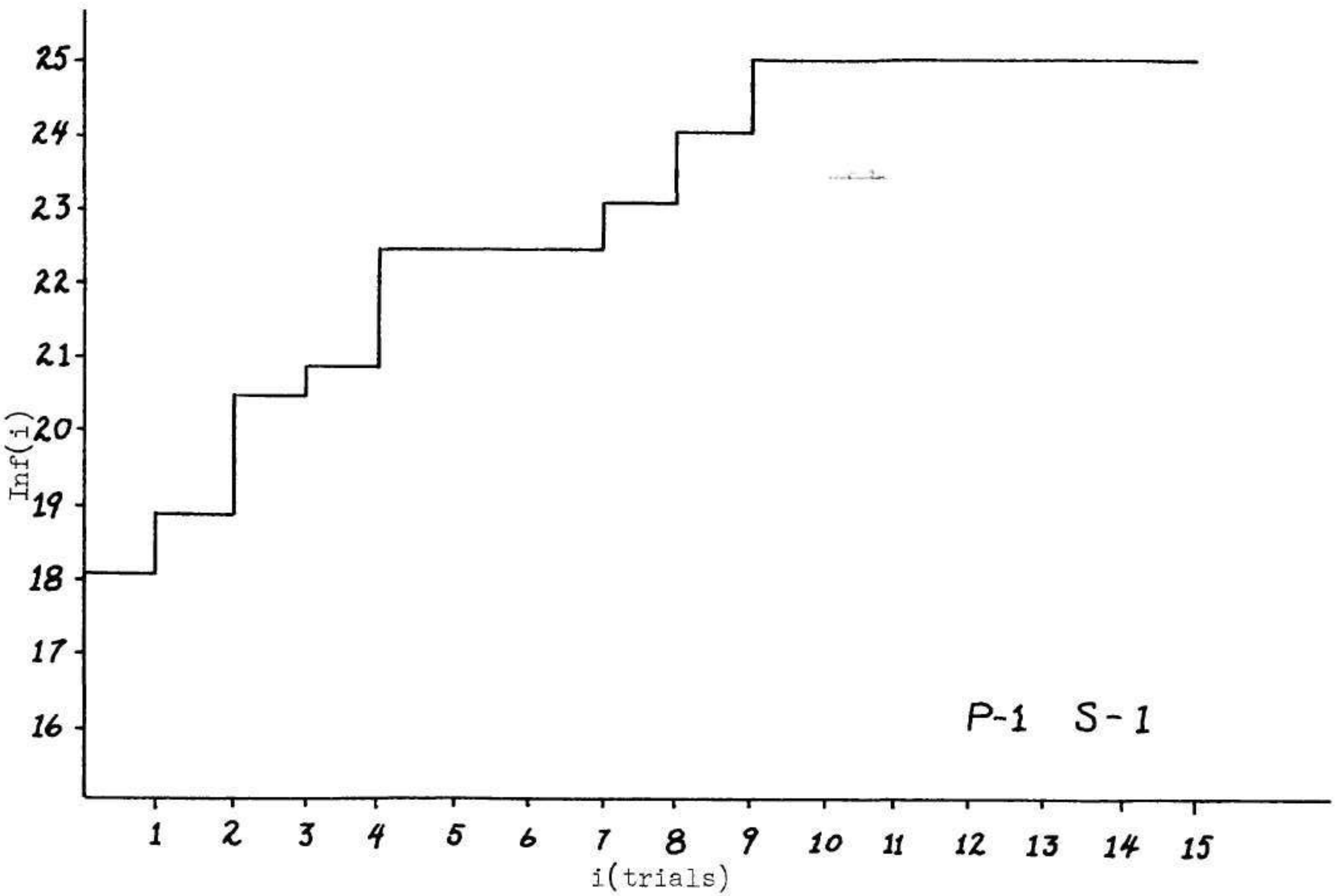


Figure 1. Information Trace for P-1.

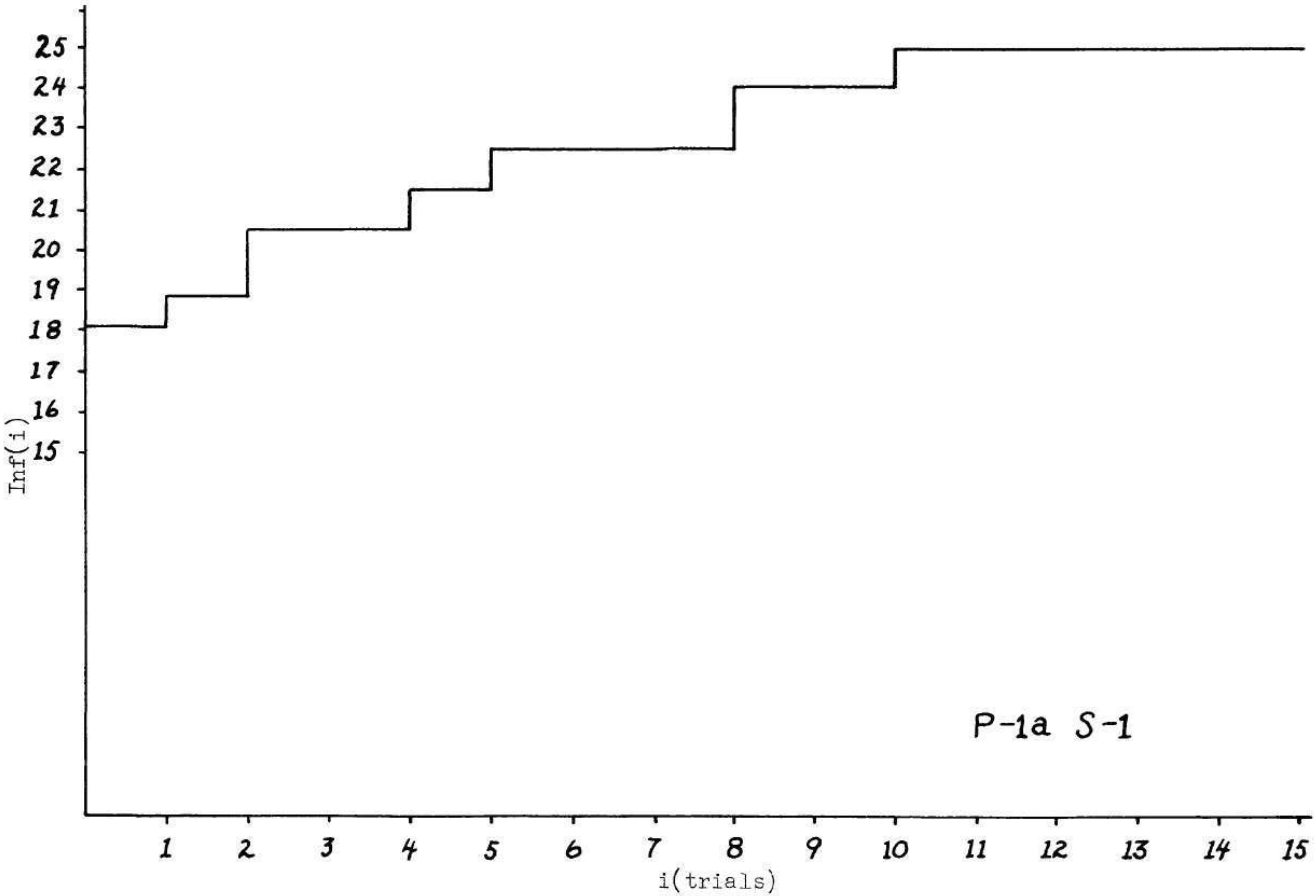


Figure 2. Information Trace For P-la.

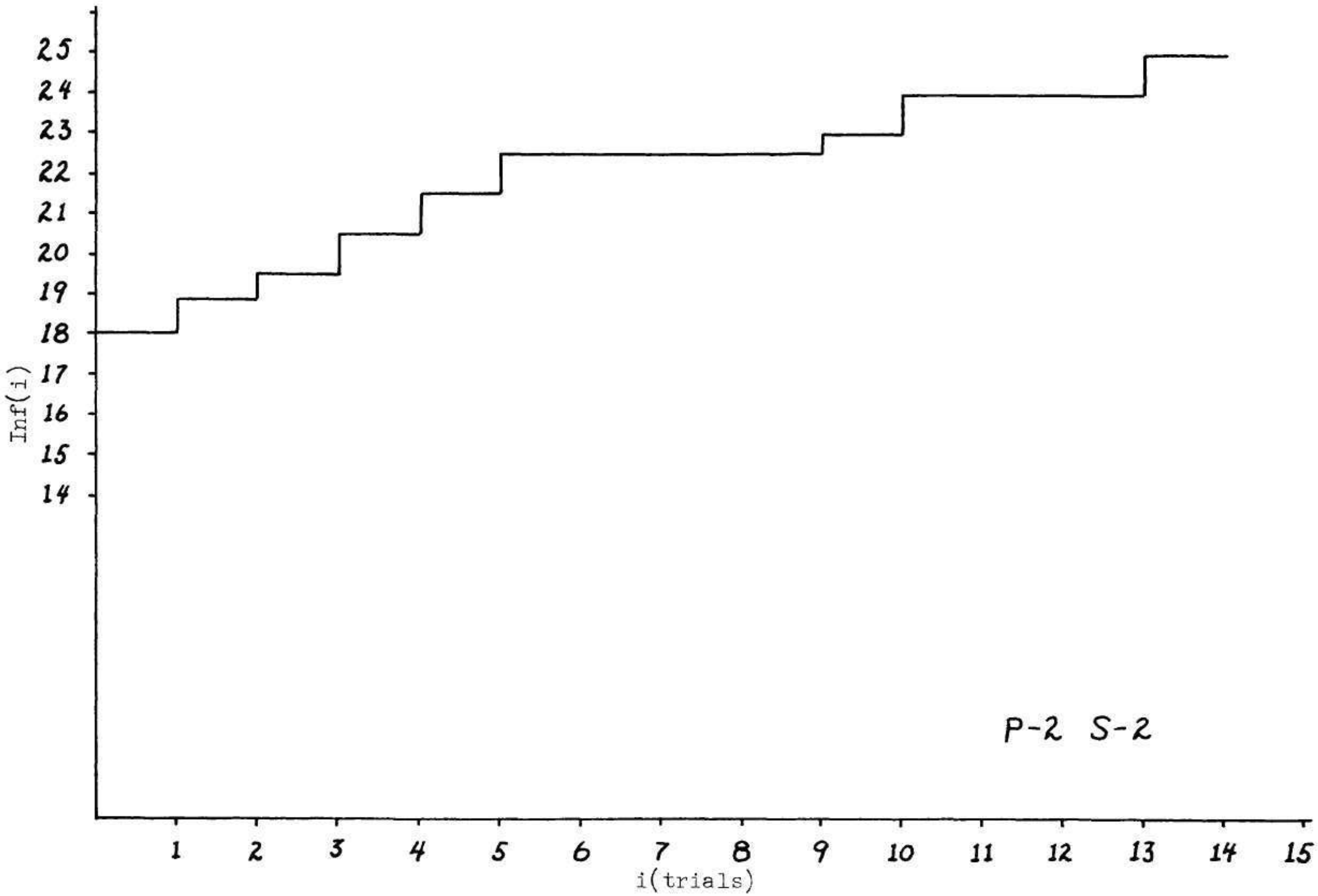


Figure 3. Information Trace for P-2.

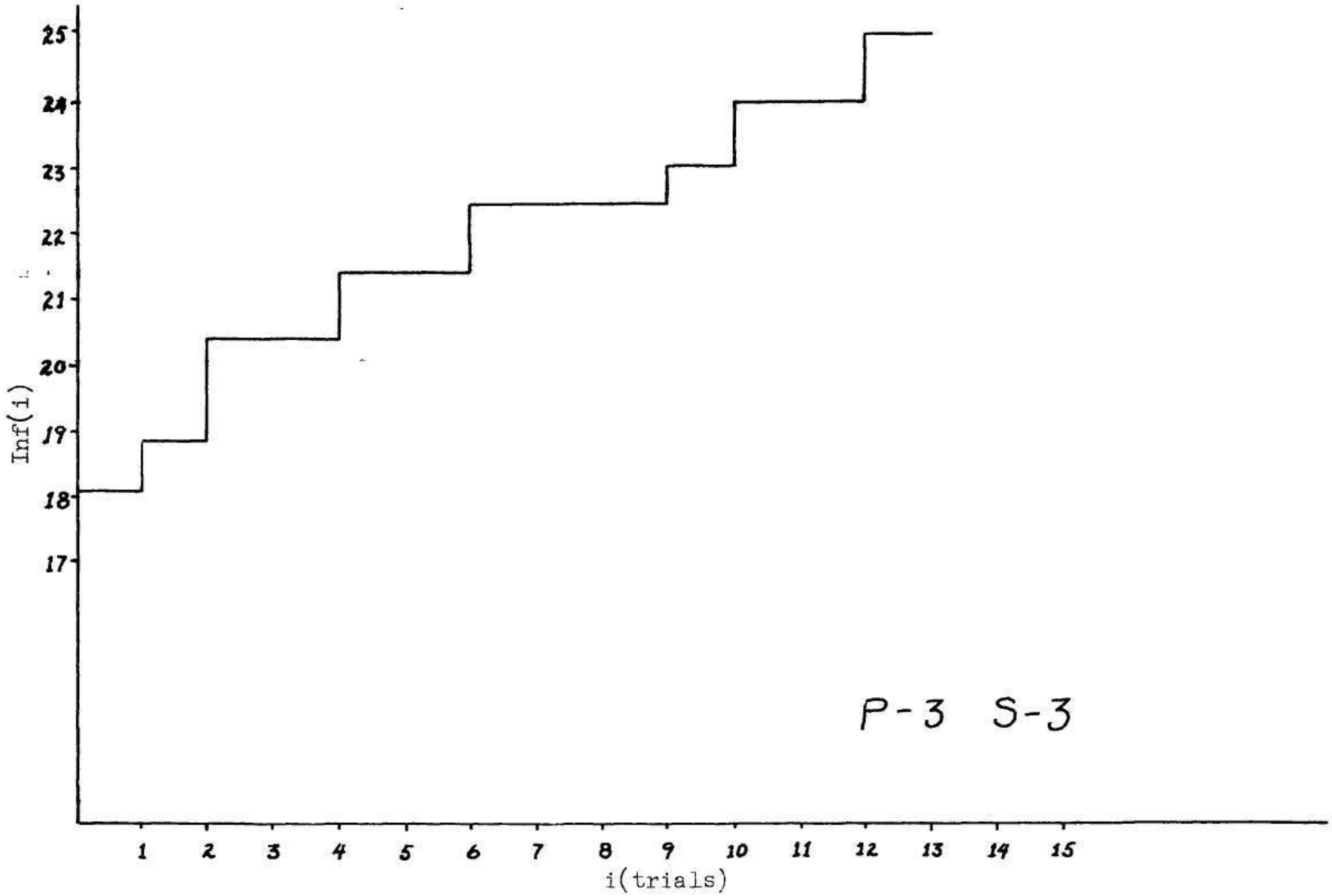


Figure 4. Information Trace for P-3.

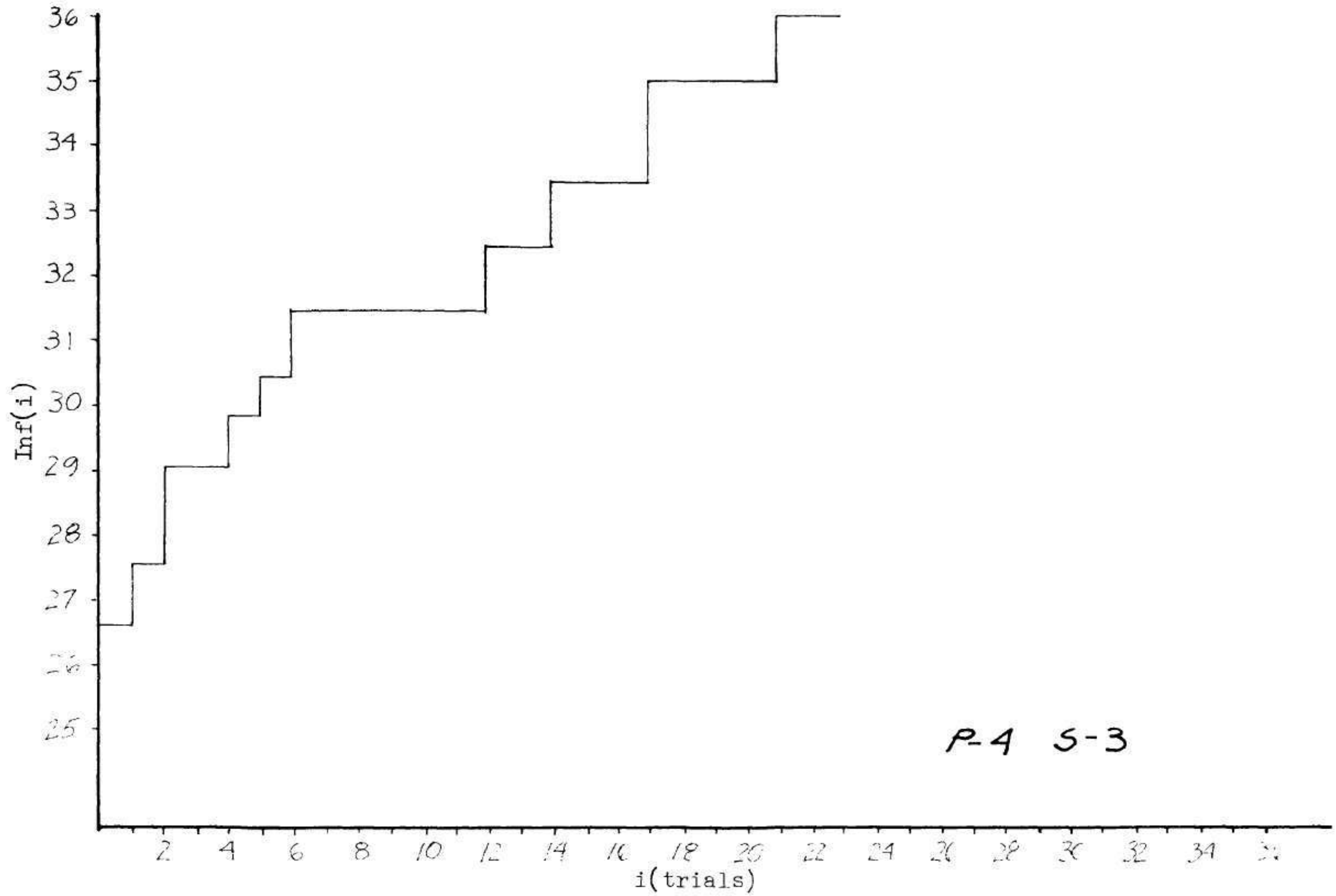


Figure 5. Information Trace for P-4.

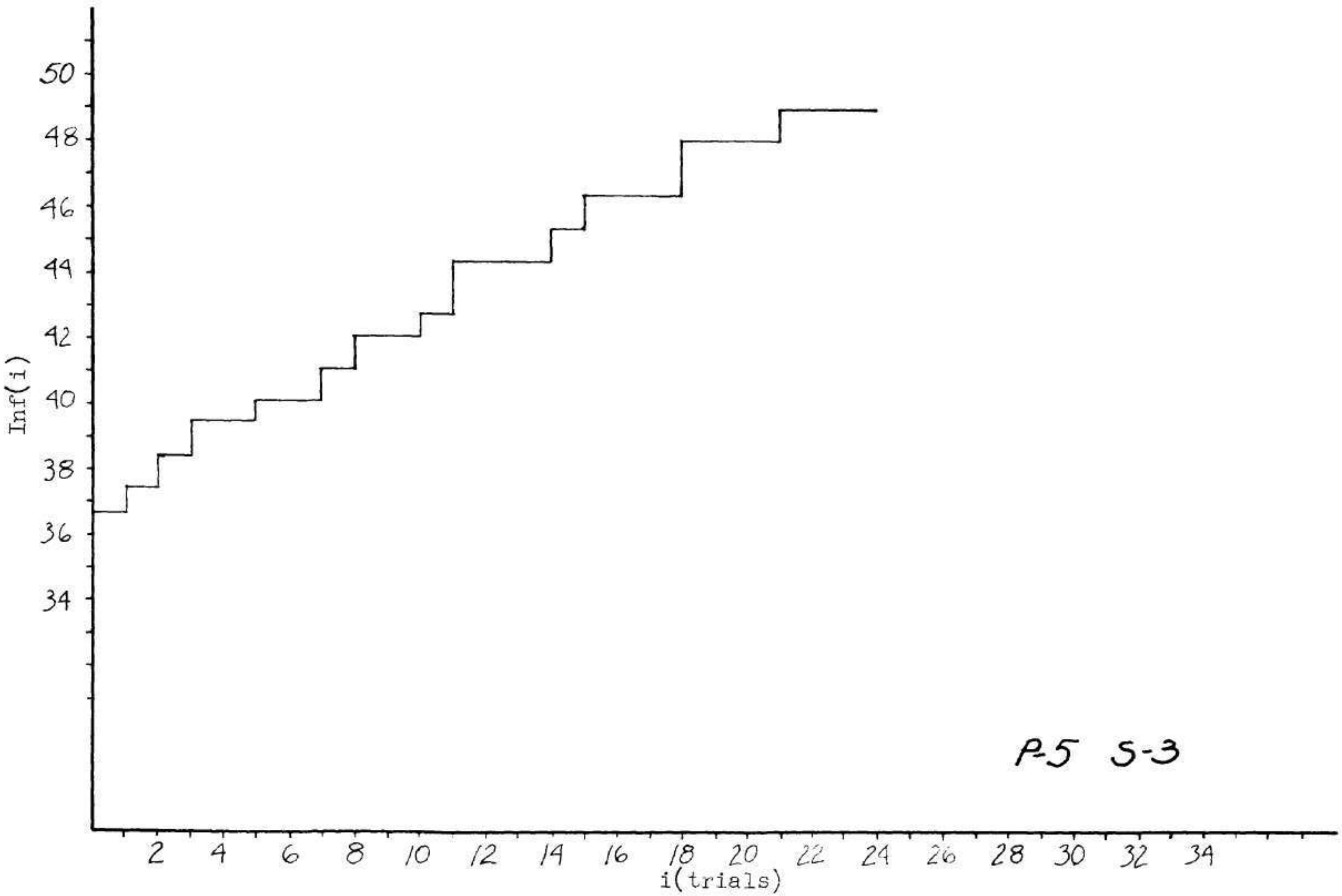


Figure 6. Information Trace for P-5.

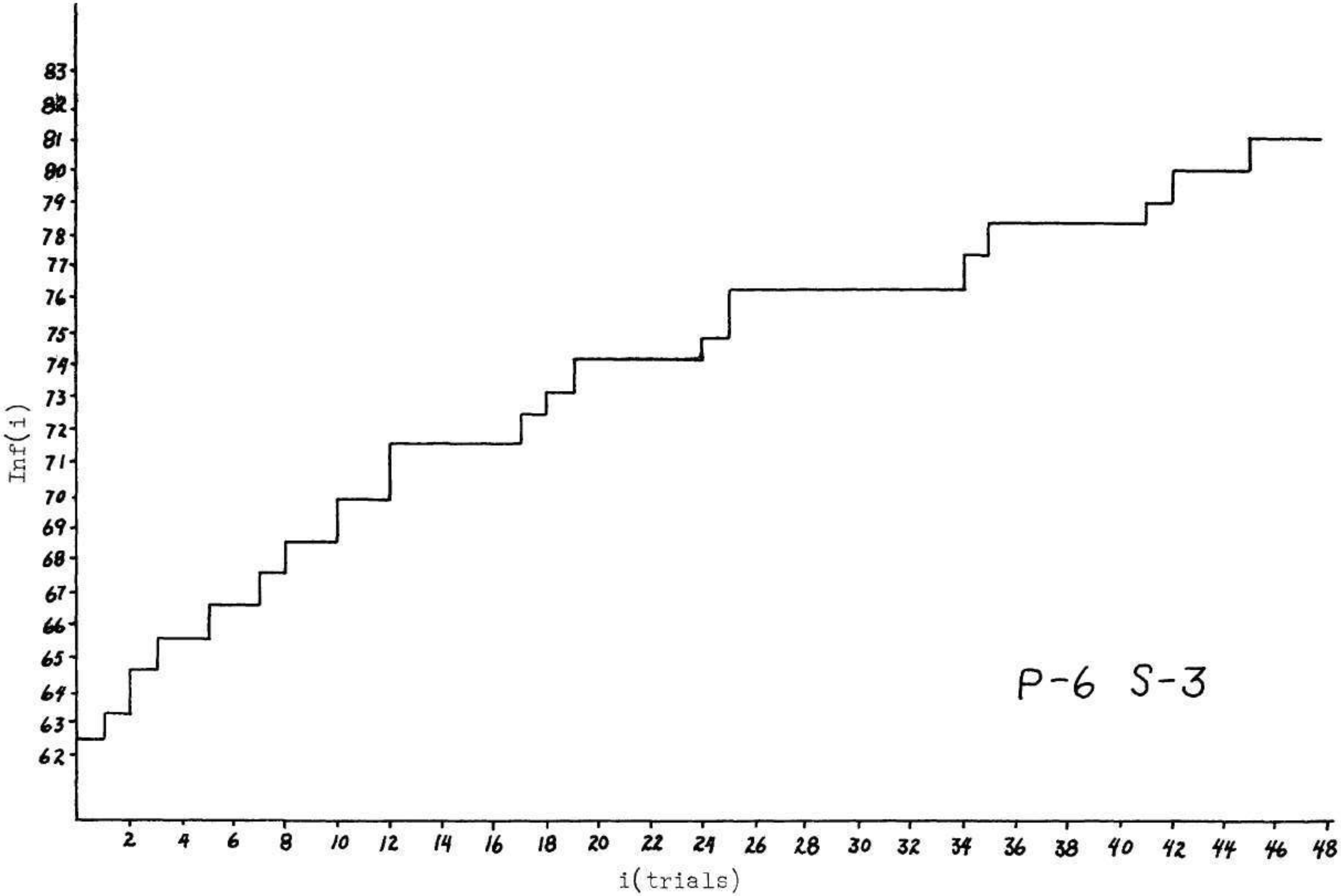


Figure 7. Information Trace for P-6.

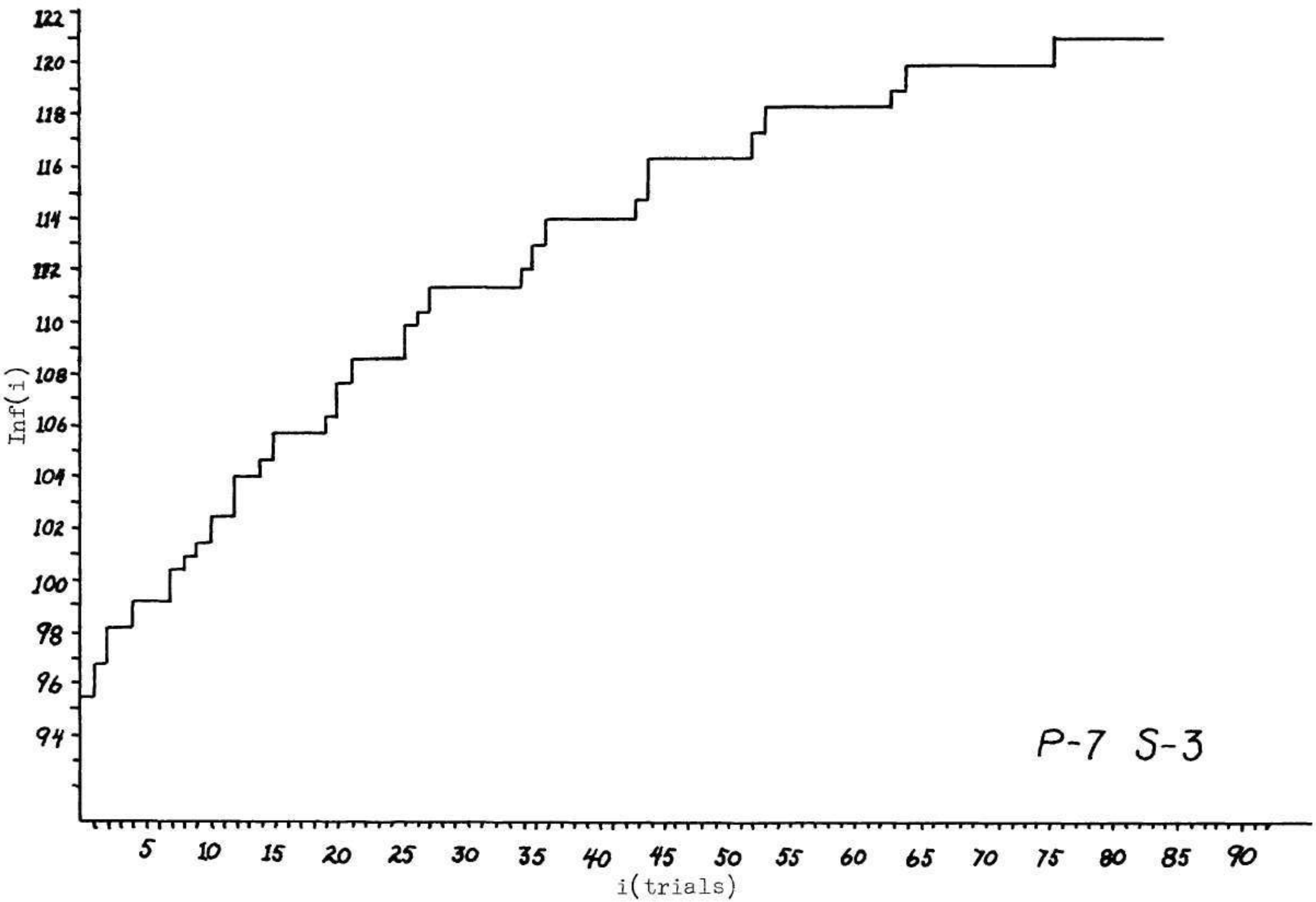


Figure 8. Information Trace for P-7.

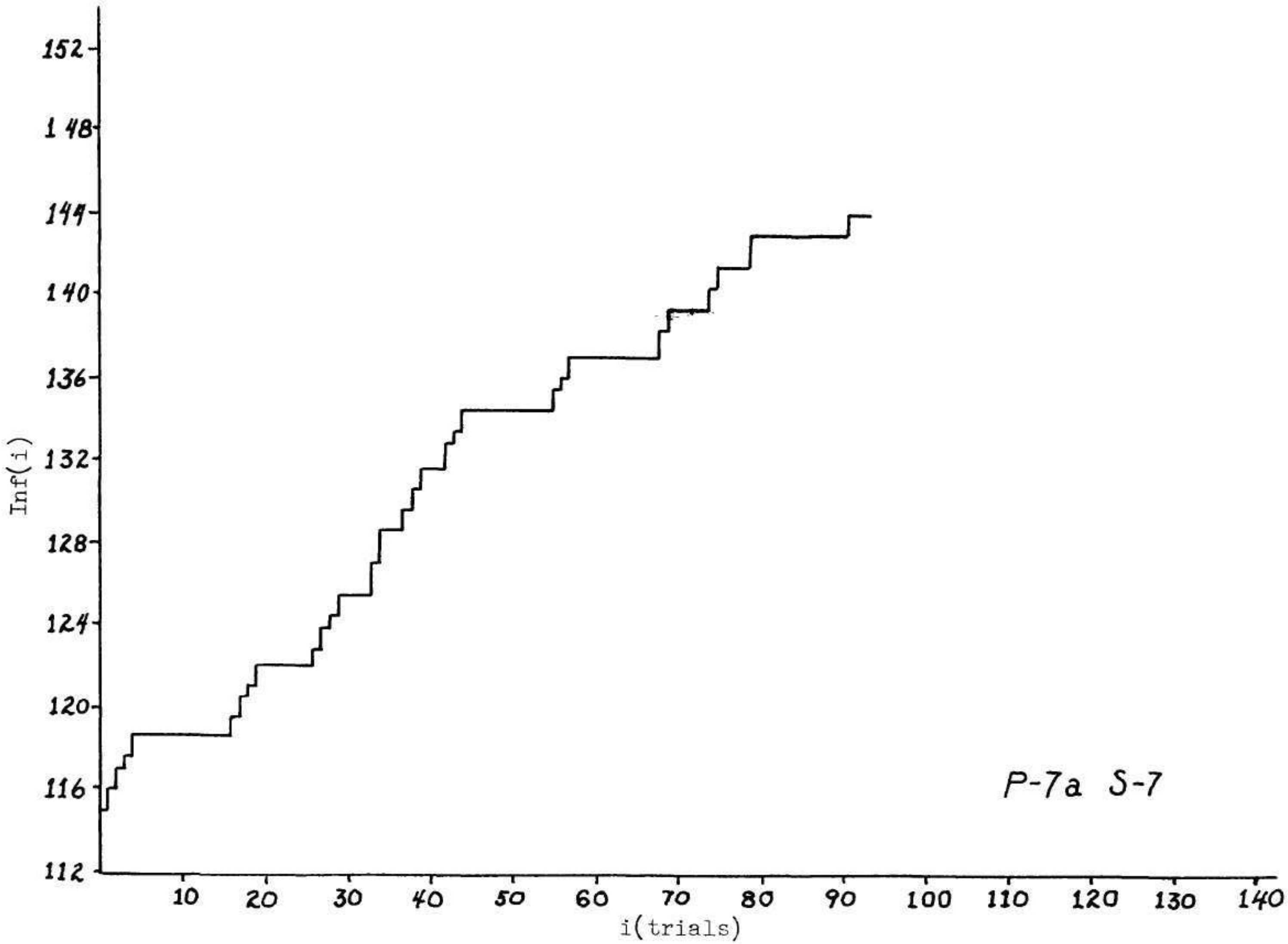


Figure 9. Information Trace for P-7a.

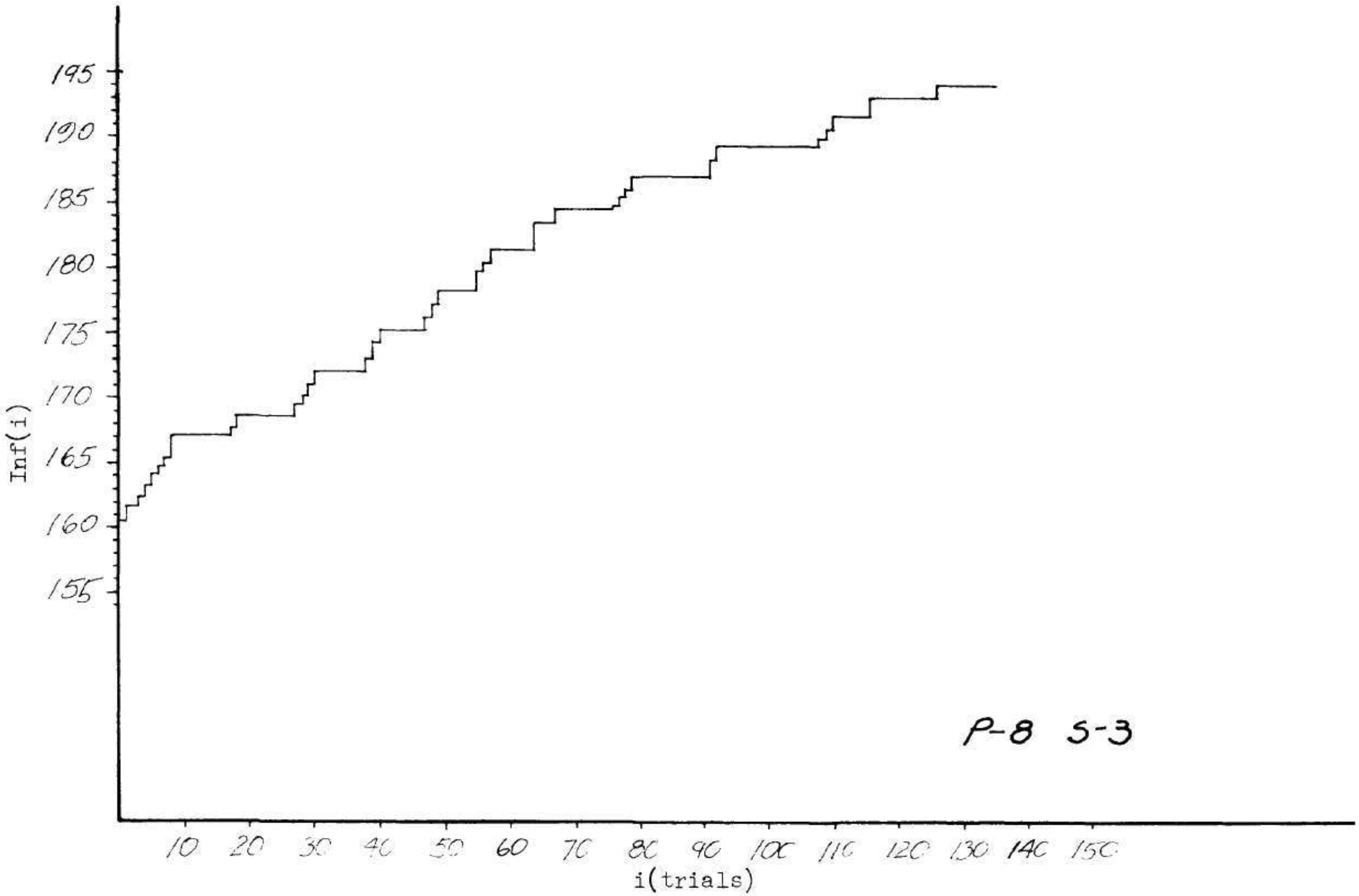


Figure 10. Information Trace for P-8.

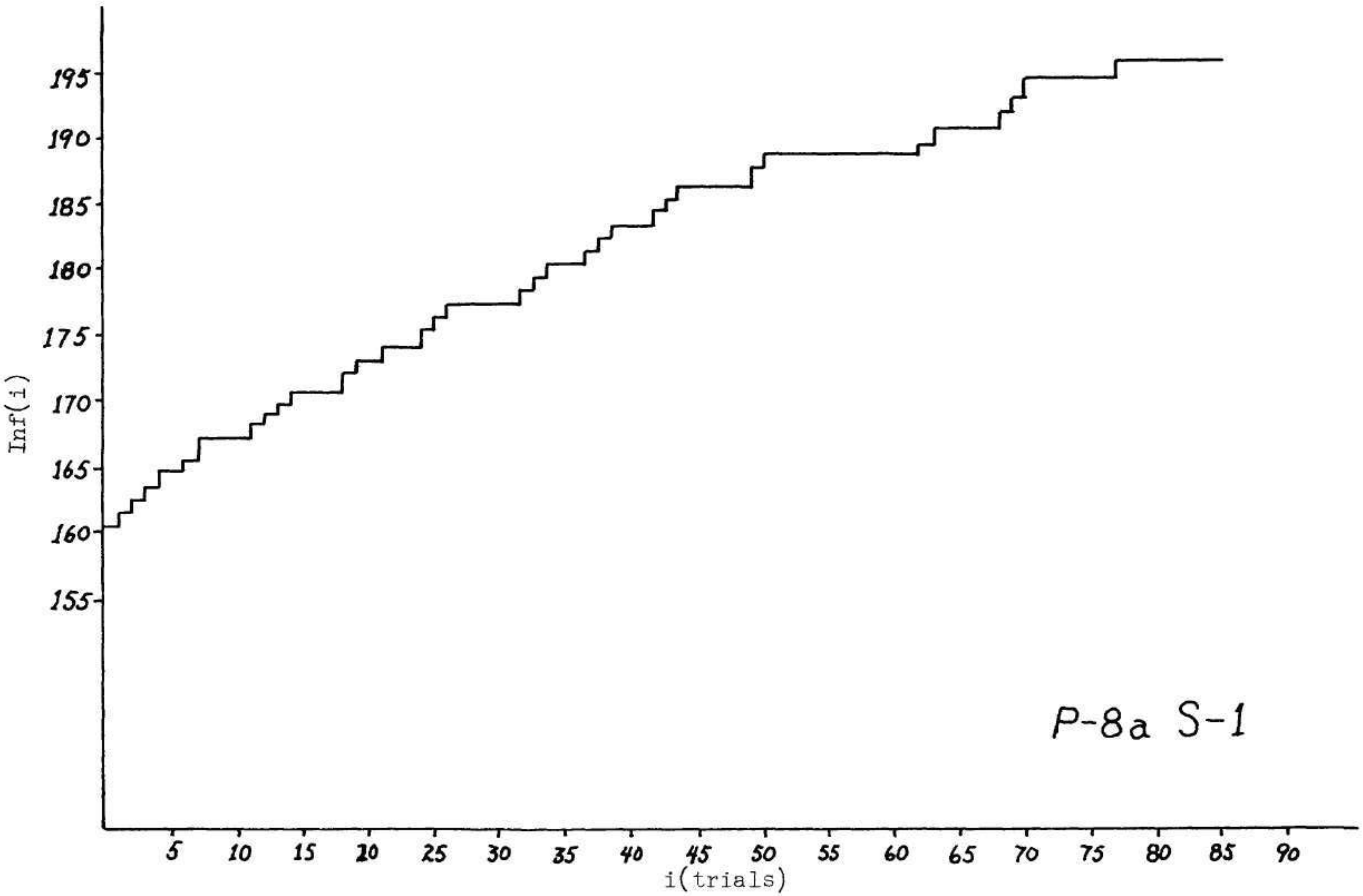


Figure 11. Information Trace for P-8a.

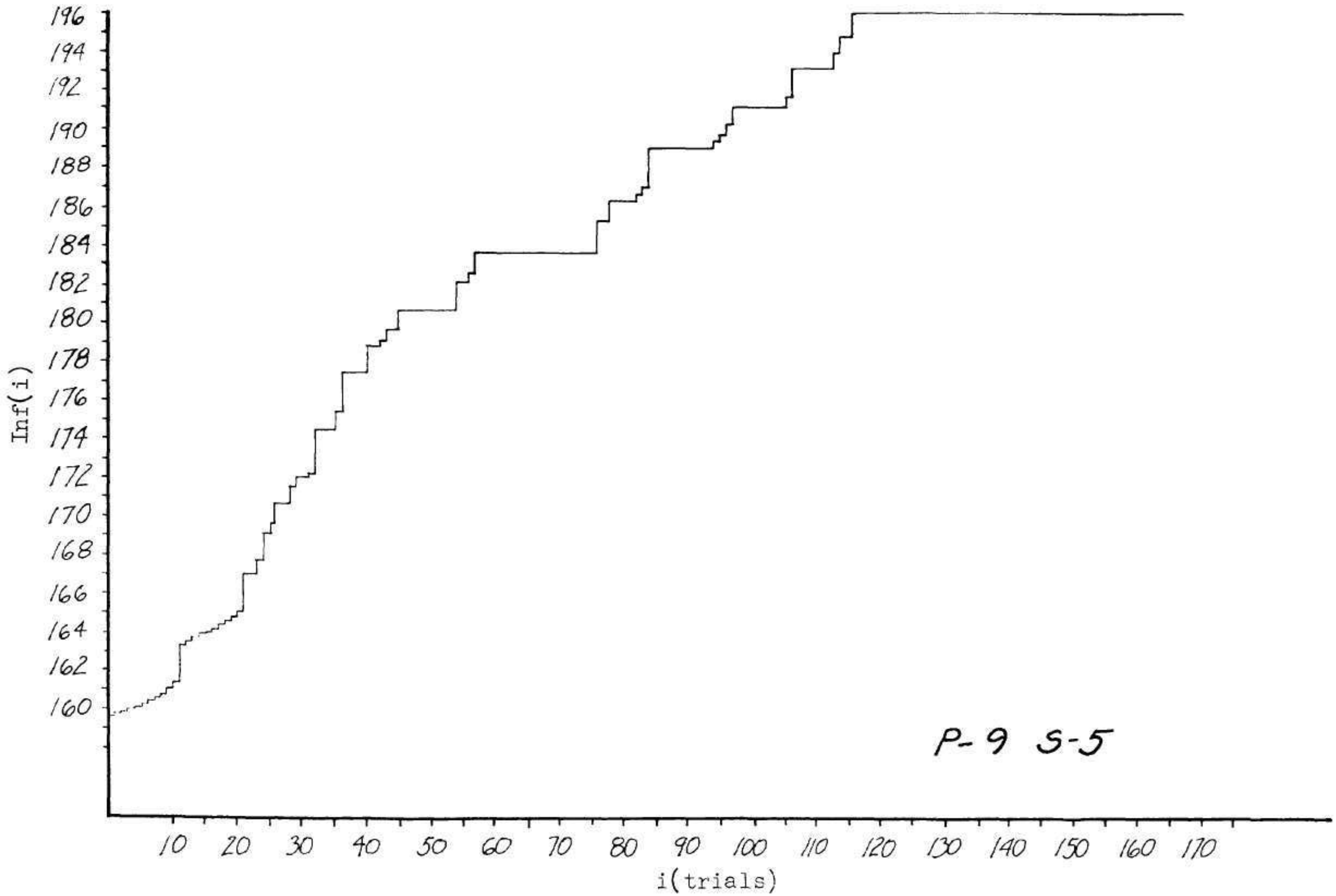


Figure 12. Information Trace for P-9.

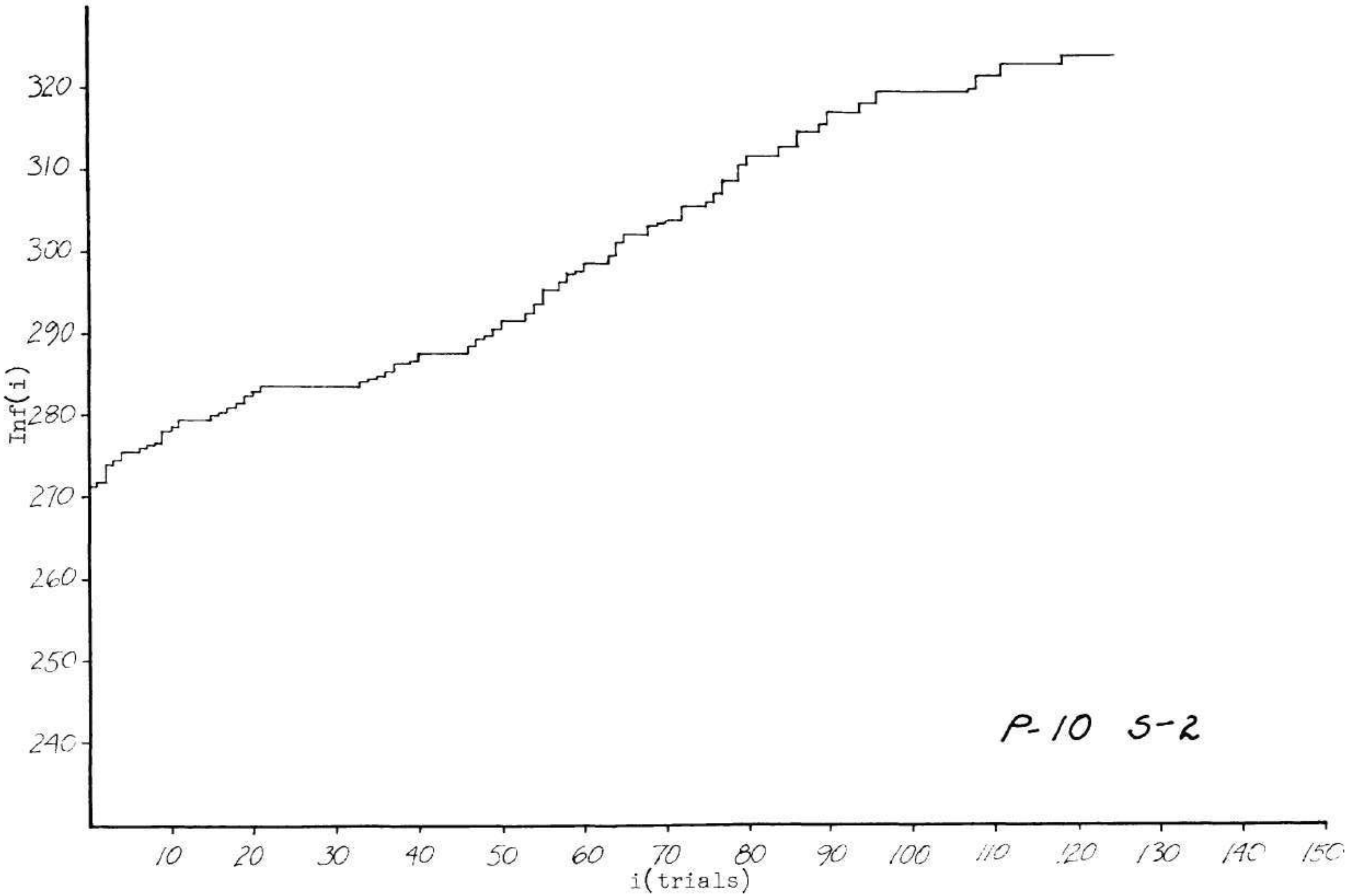


Figure 13. Information Trace for P-10.

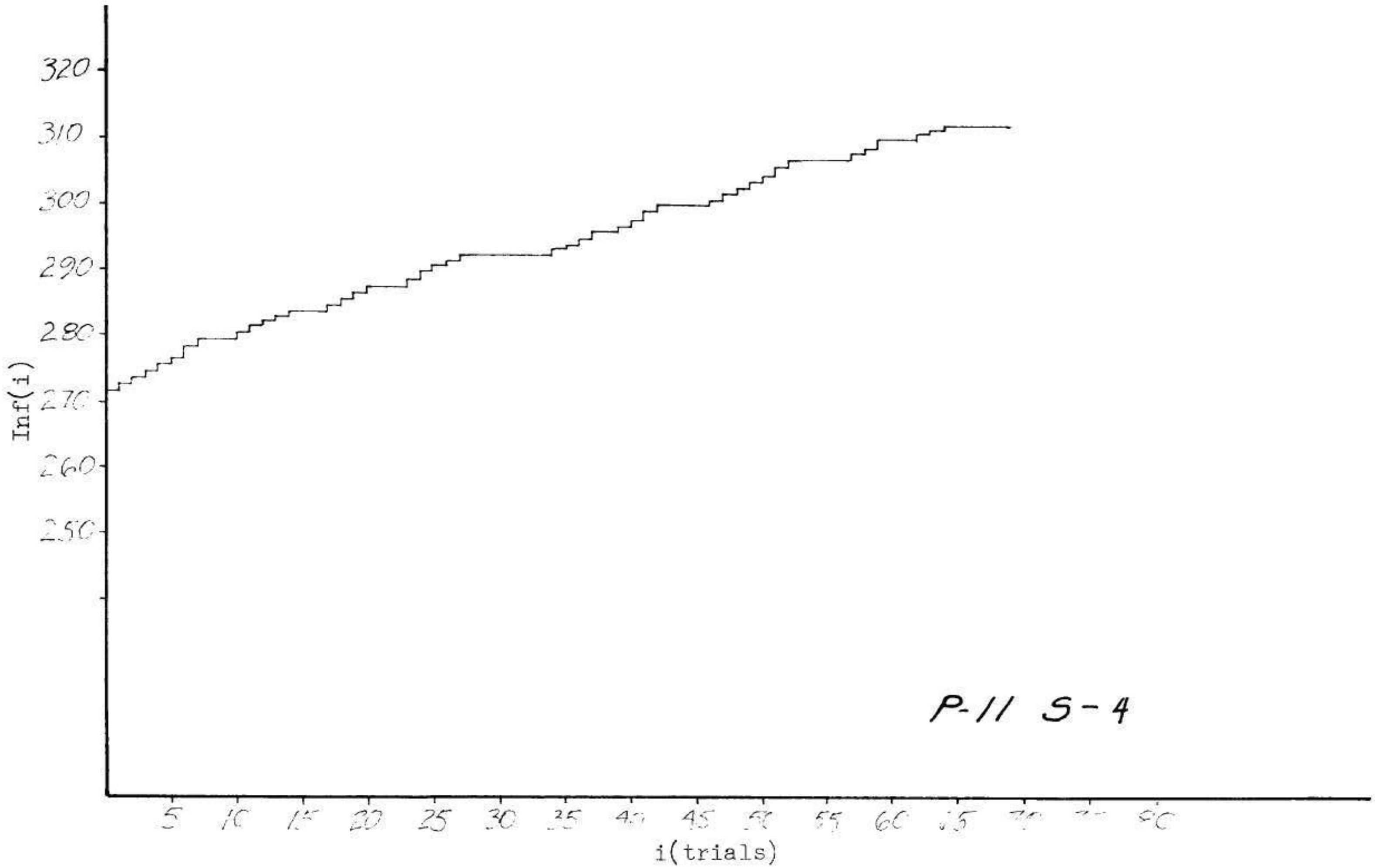


Figure 14. Information Trace for P-11.

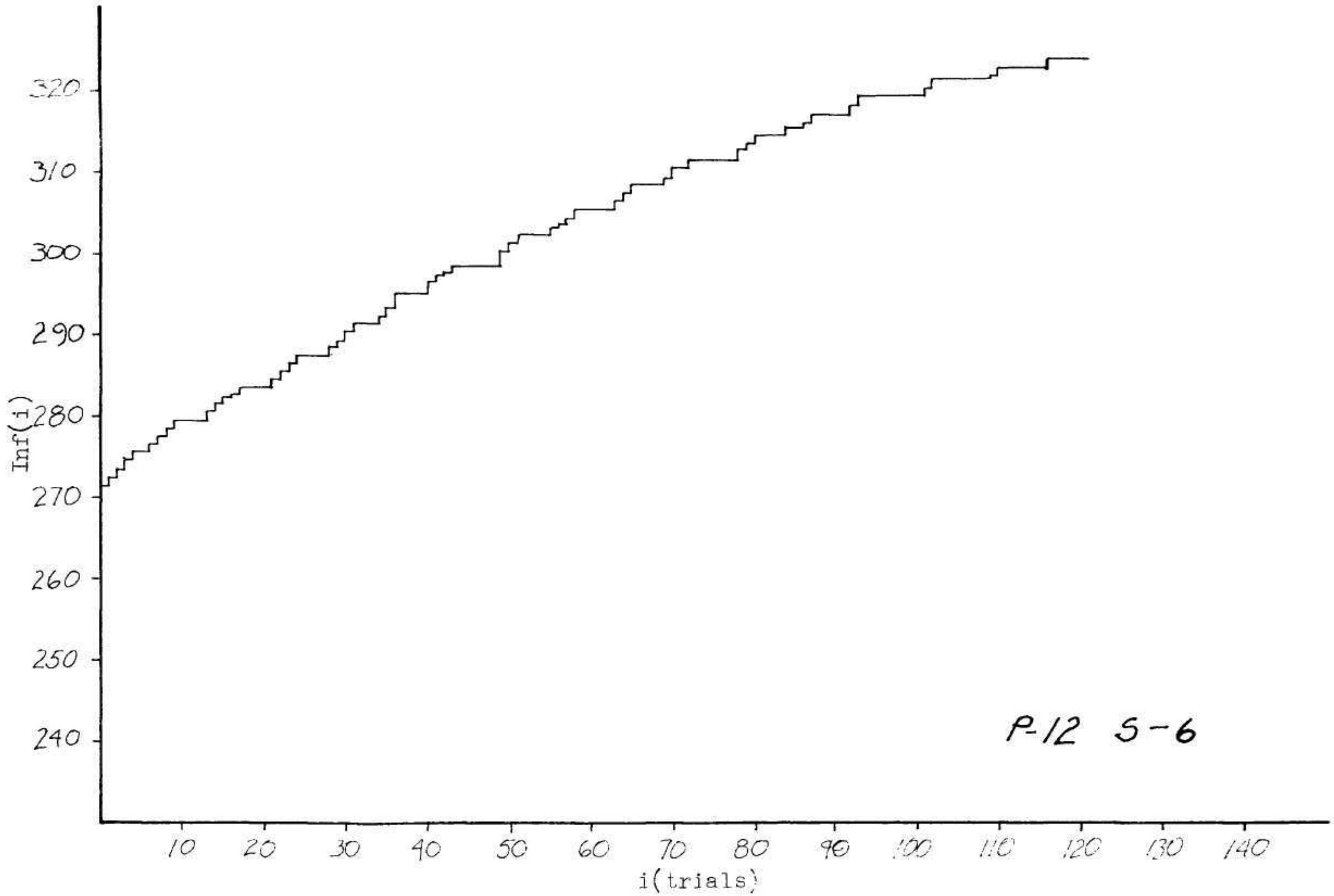


Figure 15. Information Trace for P-12.

# CHAPTER III

## MICRO-ANALYSIS OF PROTOCOLS

It is not claimed that the semantic information measure being utilized operates independently of a psychological model of information users. Such a claim would be equivalent to claiming that the SIM is universal. Rather it is claimed that there exists a psychological model of a participant in a rule learning task as an information processing system which is compatible with the assumptions of the SIM. Such a model is proposed below and includes two components: (1) a model of acquisition and (2) a model of rehearsal.

### Binary Search Behavior

In protocols of rule learning tasks assembled in Appendix I, it can be seen that a particularly efficient strategy has been utilized by several S's. This strategy can be characterized as a binary search. This is a search which partitions possible matches available for a particular $s_i \in ST$ into two equal sets with each question. In such a search if S seeks the appropriate $r_i$ which matches $s_i$ and if $r_{i_1}, r_{i_2}, \ldots, r_{i_m}$ are available (not already matched) he might ask "does $s_i$ match $r_{i_1}$, or $r_{i_2}$ or ... $r_{i_{m/2}}$"? Thus either an affirmative or a negative answer will eliminate half of the possibilities. Of course if m is not even then $[\![m/2]\!]$ responses might be mentioned in the question, where $[\![x]\!]$ is either the "round up" or "round down" function.

In order to analyze S's behavior relative to the binary search we can make what we might call a subgoal analysis. Let us suppose that S has a goal which is the rule to be discovered in the task, and that this goal consists of all possible state descriptions in P.S. This is in agreement with Newell and Simon's [1972] p. 74 definition of a problem: "Given a set U, to find a member of a subset of V having specified properties (called a goal set, G.)." Here the problem is the task presented by the P.S. and the unit goal set is the rule.

We can further consider subgoals for S. For each $s_i \in ST$, there are only certain allowable responses $r_i \in R$. (Those are exactly those $r_i$ such that no other $s_j$ has been found to match $r_i$ if the rule is one-one). Therefore, the goal consists of a conjunction of subgoals where each subgoal requires discovering which $r_i$ is matched to a particular $s_i$.

Any question that S asks about $s_i$ which is not a rehearsal question provides him with information which leads toward subgoal i. We will look at several lines of protocol Number 7. in a particular way. We will use the identity of the $s_i$ being asked about (hence the subgoal) at the left margin. We will use a bracket under a response list to indicate the range of a disjunctive question. We will use a dotted line to indicate that a disjunctive question does not include all items included in the bracket but only those indicated by vertical markers. We use the number of the question at the left of the bracket and the response to the question within the bracket. Also, all response items not available for the stimulus because of having been previously matched are circled.

From such a display in Figure 16 several things about S-3's strategy are clear. First, in all cases a subgoal is completed in sequential trials interrupted only by rehearsals. Subgoals are achieved sequentially. This sequential achievement of subgoals can of course be observed quickly in all the protocols by simply scanning the range of questions. It is of some importance, however, because the commonality observed in this matter has suggested asking an S to perform tasks without sequential subgoal achievement. The S was highly inefficient and committed a large number of errors. His introspective report was that this procedure though informationally equivalent was much more difficult than the sequential subgoal achievement.

Of more immediate interest is the observation of a search strategy much like a binary search. It is of considerable interest then to observe that to some extent this strategy is observed not only in S-3 but also in all other S's. S-5 began in the first 21 trials of P-9 to follow a search strategy of a different nature. At this point he stopped and asked for an additional reading of instructions concerning compound questions and proceeded from that point to follow an approximation to the binary search.

The discovery that many S's follow a highly regular and effective search strategy and that this strategy produces performances with relatively little error, suggests that these S's are behaving as information processing systems in Newell and Simon's sense [1972].

In order to look at our data according to techniques similar to those of Newell and Simon we must first consider the problem space adapted by S's. In Chapter I it was briefly argued that the disjunction

P-7 S-3

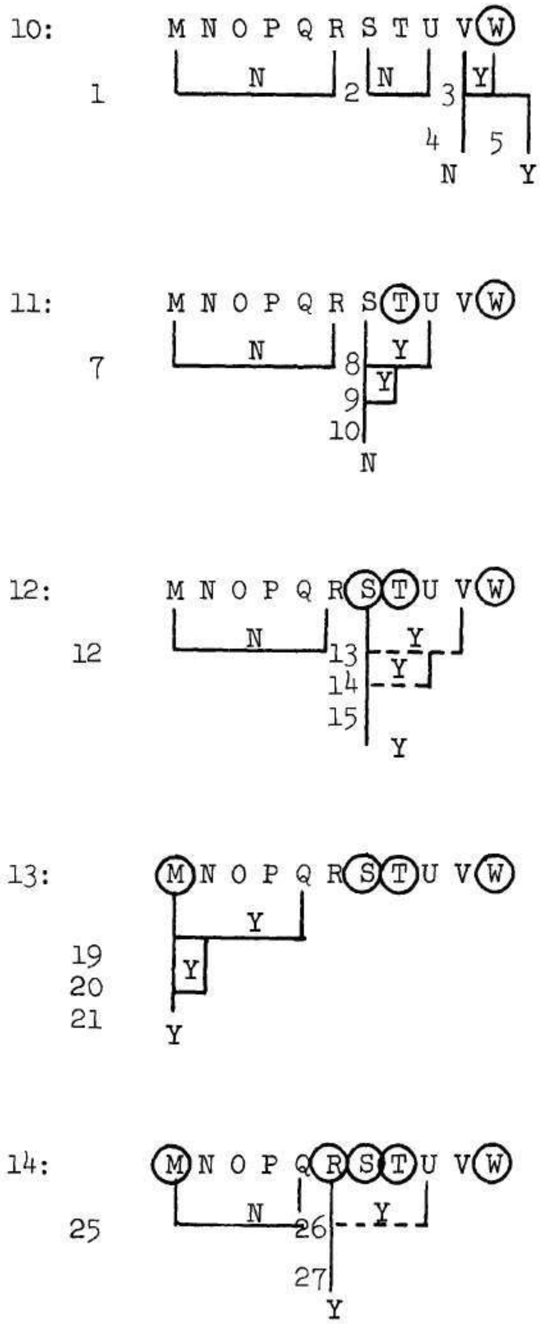


Figure 16. Subgoal Episode Analysis.

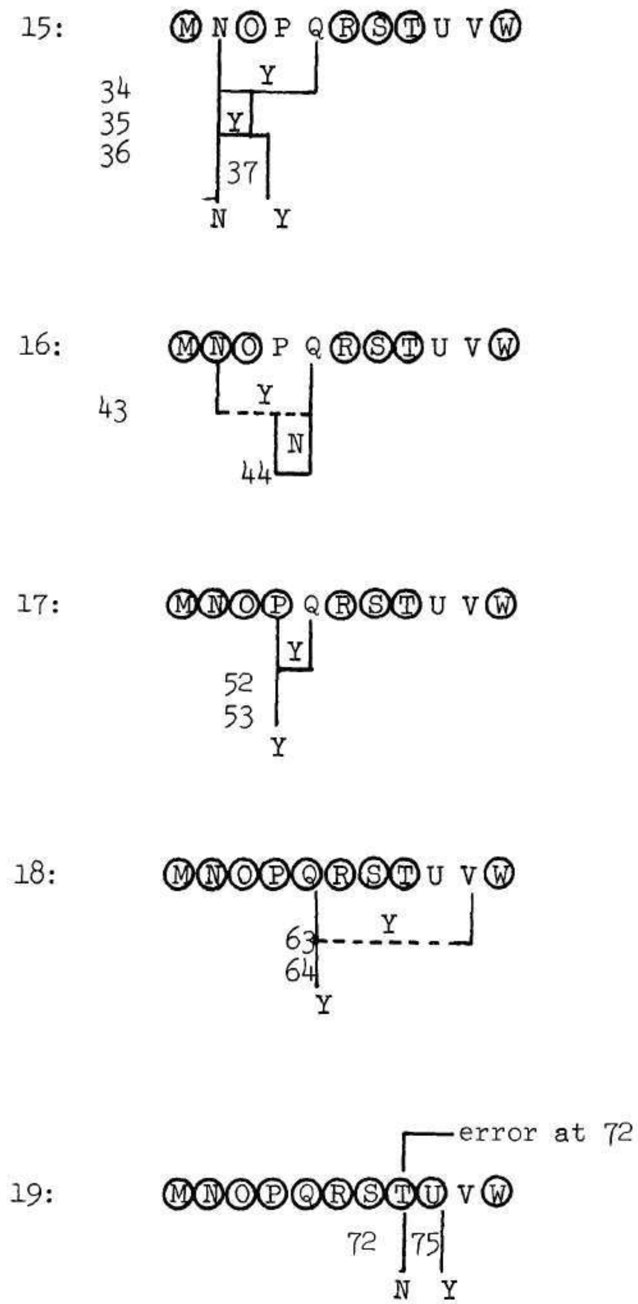


Figure 16. Subgoal Episode Analysis. (Continued)

of state descriptions compatible with the instruction about the nature of the rule to be learned is a description of the problem space for S. This sense of problem space is more restrictive than that of Newell and Simon. Newell and Simon, p. 810, would include in the description of a problem space all of the following:

1. A set of elements: these are symbol structures which are representations of states of knowledge about the task.
2. A set of operators: these are information processes by which S can move from one state of knowledge to another.
3. An initial state of knowledge that S has about the task.
4. A problem: a set (possibly singleton) of designated knowledge states.
5. The total knowledge available to S: this includes contents of primary, secondary, and external memory as well as path information, etc.

What we have designated as P.S. is most closely allied to 1 above. An alternative could be taken as follows. The disjunction of all the state descriptions in $\mathcal{L}_N^M$ could be taken as 1, while what we call P.S. could then be 3. We choose, however, to take all disjunctions of subsets of the disjuncts of P.S. as the set of knowledge states and take the initial state to be P.S.

The set of operators are the $Q_i$ available to S, the answers to the $Q_i$, and the information processors S brings to bear upon the $Q_i$ and their answers. The description of these information processors will occupy us below.

Discovering how S accesses E.M. is a very difficult process.

This is true because of the difficulty of calibrating eye marking cameras and other such devices but is also true because of the difficulty of accounting for the interaction of E.M. with other memories. A procedure for dealing with this problem in a relatively straight forward manner is discussed in the section on proposed research in Chapter VI. By utilizing only minimal E.M. we allow an investigation of the participation of STM and LTM in S's behavior in a much more direct manner than that of Newell and Simon.

We assume that a minimal set of information processors that are necessary to account for the binary search strategy shown above includes:

1. The ability to store sets in STM.
2. The ability to mark elements of sets or alternatively store subsets of other sets in storage.
3. Ability to perform unions (or additions) of sets in memory.
4. Ability to take relative compliments (or substraction) of sets in memory.
5. Ability to make comparisons on cardinality of sets in memory.
6. Ability to partition sets in memory.

These processors are elementary information processes (e.i.p.'s in Newell and Simon (p. 20). The program which organizes these(e.i.p.'s) in such a way as to account for an s's behavior we take to be S's strategy.

To construct a strategy program we shall let:

1. S be a set of Stimuli
2. R be a set of Responses

3. M(S) be a set of Stimuli marked as processed.
4. M(R) be a set of Responses marked as processed.

An information processing model based on this set of assumption and e.i.p.'s can be constructed to account for observed acquisition behavior.

## Strategy Program

The programs which operate on the problem space to account for S's acquisition strategies are divided into three levels and in one case four levels. These are (1) the goal program; (2) the subgoal program; (2) the subgoal program; (3) the response set selector program. The fourth is a program which was needed to account for one subject's strategy and will be discussed separately. This structure was chosen as a result of two factors. First, introspective data from participating S's indicate that the goal of learning the rule is subdivided into subgoals associated with each stimulus item in a very natural way. This is borne out by the data which indicate S's almost invariably seek information about the rule to be learned component by component, finding a match for $s_i$ before ever asking any information about $s_{i+1}$. This tends to verify the validity of a goal for the task which is subdivided into subgoals for each stimulus item. The second reason that this structure was chosen is that there is a commonality to certain aspects of the acquisition by all subjects and other aspects that are much more particular to individuals. As mentioned above the selection sequentially of stimulus items to ask about is almost perfectly common to all participating S's. The type of questions generated about the selected stimuli, however, displays much less commonality in this aspect of acquisition. For this reason a level of distinction is made between a goal program which selects a stimulus item or terminates and a subgoal structure which generates questions about the stimulus passed to it by the goal program. The Goal program is common to all protocols and subgoal programs are unique to various question strategies. The variation in these subgoal strategies are not great but these differences are captured by programs which account for them.

The reason for separating the response set selecting program into a third level is less firm than the reason for the separation of Goal from Subgoal. The Goal and Subgoal programs are supposed to reflect to some extent the psychological processes that they represent. The Select program is not. For this reason it is placed external to the other levels. The Subgoal program places bounds on the subset of responses to be asked about and select is only a mechanism for generating response subsets from these bounds and might in fact be omitted as not contributing to any additional understanding of S's psychological processes. It is included only because these programs can be very easily constructed in any of several programming languages and such an endeavor would require a mechanism such as select.

## Goal

The claim that this program holds for all observed strategies is valid only if small deviations are permitted. For example, in P-4 S-3 asks "does 5 match w?" and he receives a response of "no." He then asks "does 6 match w?" Here it seems that S is pursuing a match for w (a response item) before leaving it. This appears to be at

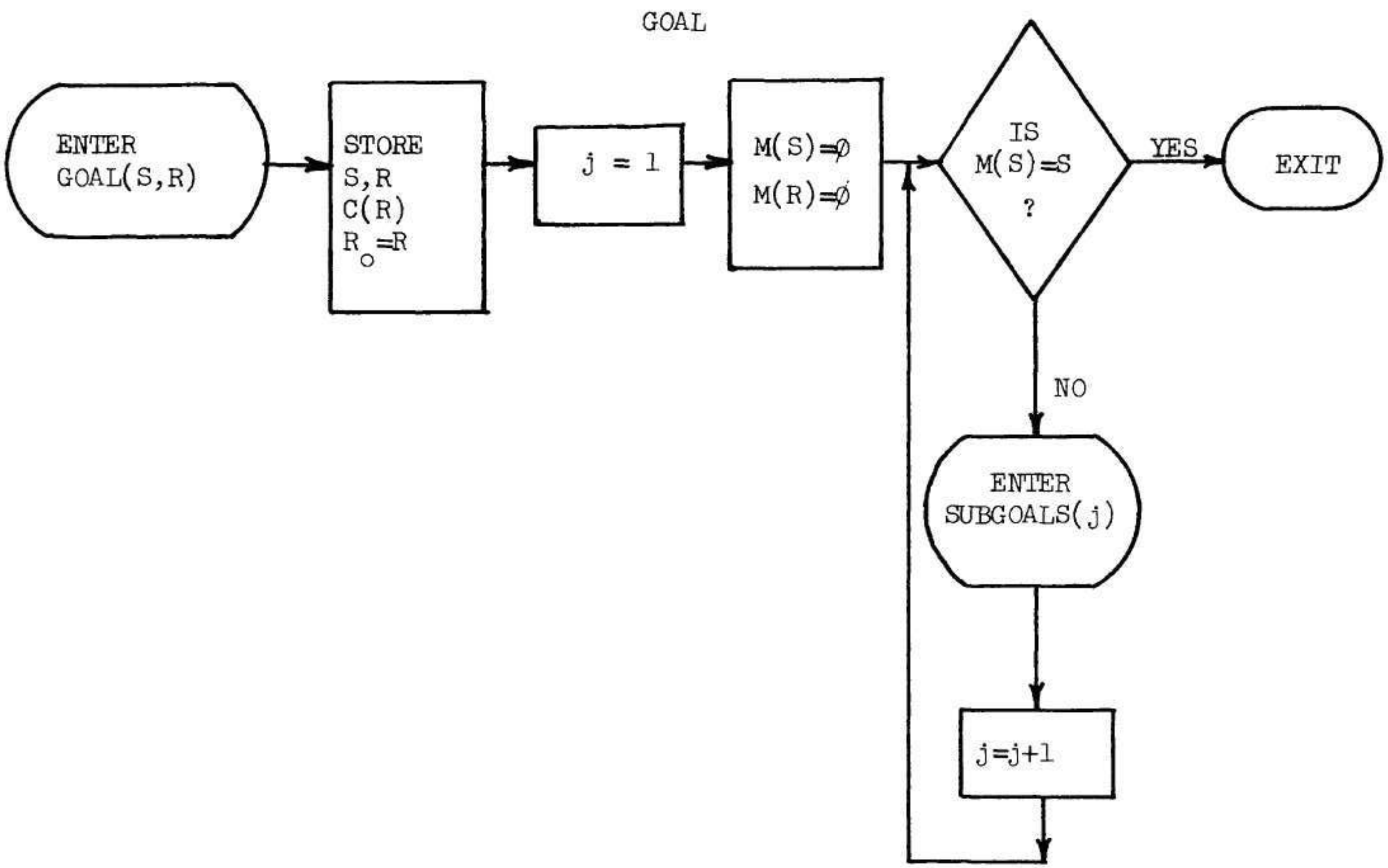


Figure 17. Goal Routine.

variance with earlier trials in which he continues questions about a stimulus item until a match is found. See trials 7 through 10 for example. This exception is not great if it is an exception at all since at line 21 there is sufficient information to imply a match for 5. Knowing that 5 does not match w, S knows that it does match s if he utilizes all available information (as he does elsewhere; note that IAS = AIS for trials 6, 10, 11, 15, 16, 19, 20 and 23).

Goal takes as arguments the Stimulus and Response sets, S, and R, as provided by the instructions. These along with the cardinality of the responses are stored in memory. Also a set $R_o$ which is to serve as a set of unmatched responses is initialized as R in memory. A counter which will indicate the subgoal to be processed is initialized. The sets of stimulus and response items marked as matched are initialized as empty ($\emptyset$). The decision to proceed to a new subgoal or to terminate is based on whether or not all stimuli are marked as matched. This could just as well be based on C(S) or C(R) (C(S) = C(R) for these tasks); however, introspective data indicate this choice to be the more natural one. Exiting the goal program indicates that S has completed the task.

## Subgoal

Subgoal is particular to various strategies. We therefore begin with the subgoal program which has the most generality over participating S's. As discussed above, all S's to some extent approximate a binary search. The degree of approximation to pure binary search varies from near perfect in S-3's case to very divergent from

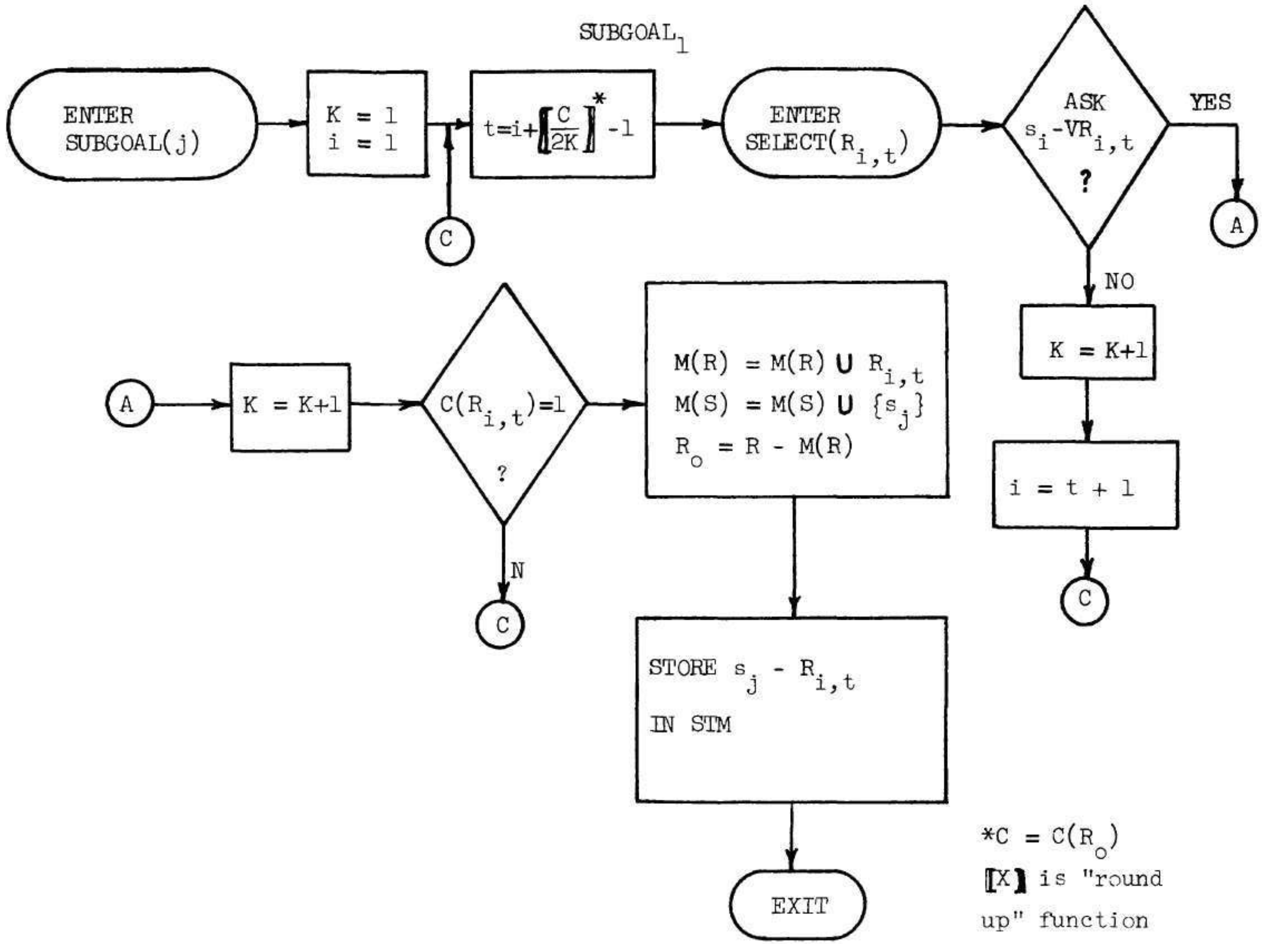


Figure 18. Subgoal$_1$ Routine.

pure binary search in S-2's case. One main difference between search strategies seems to be whether marked responses are dropped out before or after the partitioning is done. Another difference has to do with whether or not half the available responses are chosen (as in P-7, P-12, and P-9) or some smaller set is chosen (as in P-10). The sub-goal program which accounts for pure binary search will not fit the protocol of any S in any task perfectly. Each S has some deviation from this strategy even though this deviation is often quite small. It will, however, account for the strategy of several S's at a surprising degree of fit. This produces evidence that these S's process information in a highly regular way and from the point of view of a later analysis, in a nearly optimal way.

The argument of $Subgoal_1$ is passed to it by Goal. Other internal counters i, K and t set bounds on the response set $R_{i,t}$ which is ordered from Select (below). When $R_{i,t}$ is returned from Select, a question "Does $s_j$ match $r_i$ or $r_{i+1}$ or...or $t_t$?" is generated. If the answer is "no" then i and t are altered to cause Select to return another appropriate response subset. If the answer is "yes" either i and t are reset to partition the present $R_{i,t}$ appropriately or information is stored in preparation for exit according as to whether $R_{i,t}$ contains more than one response item or not. The information stored involves marking the matched stimuli and responses and entering the matched pair itself in primary memory.

## Select

The mechanism for selecting the response subset to include in

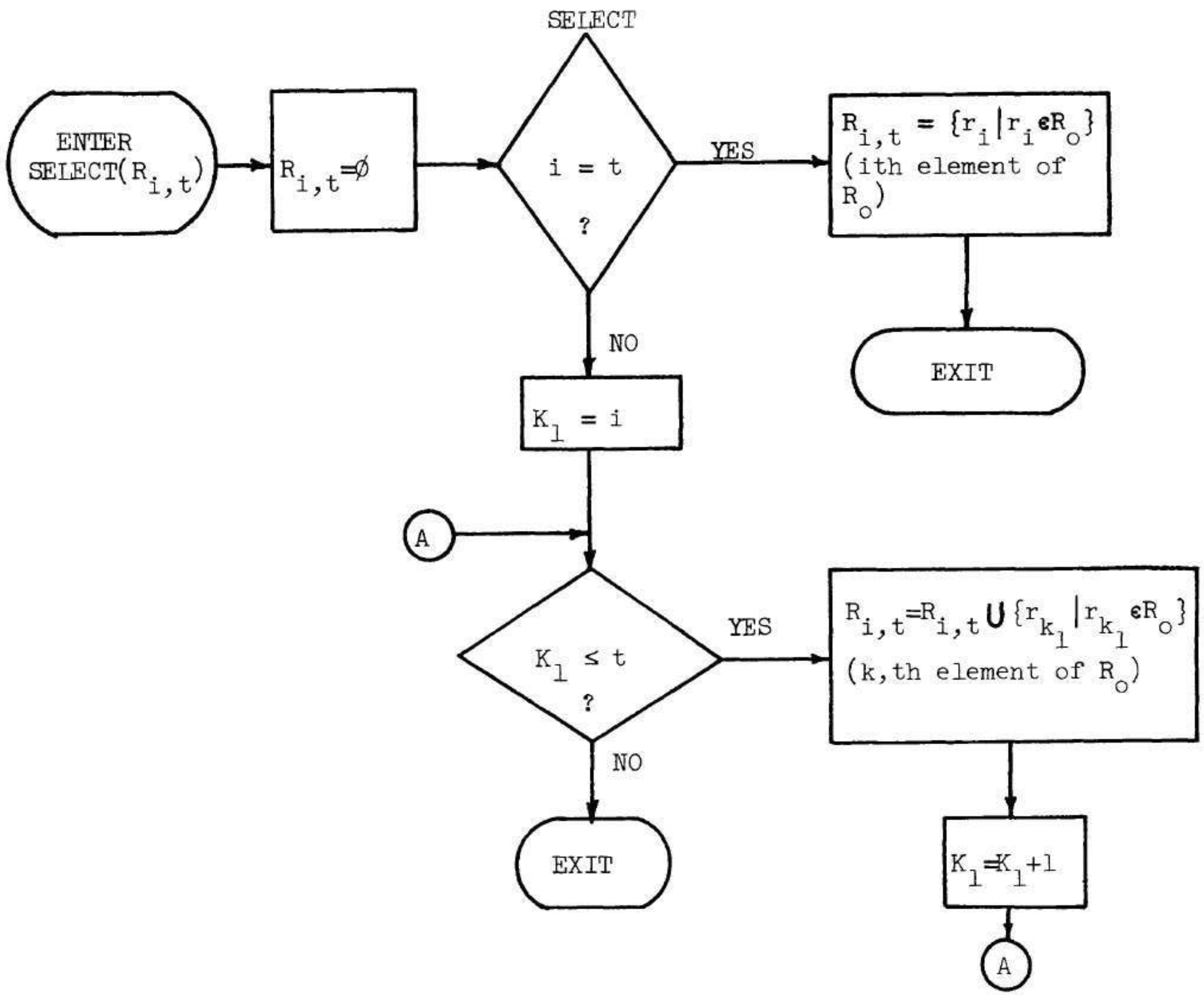


Figure 19. SELECT Routine.

each question requires bounds which determine how many response items are to be included. It also must have the capability of selecting these items either from R or from R-M(R) according to the strategy employed. In order to make Select general, the setting of $R_o$ (the set of available matching R items) to be either R or R-M(R) is done in subgoal.

If the upper and lower bounds passed to Select are equal (i=t) then $R_{i,t}$ is the singleton containing the ith element of $R_o$ otherwise a counter is used to put all elements $r_i$, $r_{i+1}, \ldots r_t$ of $R_o$ into $R_{i,t}$. This set is then returned.

With Goal, $\text{Subgoal}_1$ and Select we can construct a trace of the Strategy program for a particular task and compare it with the actual protocol of interaction with a subject in the same task. For demonstration purposes we will choose P-4, a protocol of S-3 who seems to have followed a strategy quite close to a pure binary search. In a rule learning task involving six stimulus items and six response items S-3 learned the rule in 23 trials. For 13 of these trials $\text{inc}_i$ is zero and are therefore rehearsals according to Definition 9. Even though trials 3, 10, 18, 22 are rehearsals by the strict definition we will include them in the acquisition trials for clarity. This is due to the fact that they contain rehearsal of information that was discoverable by inference and would not explicitly occur in a protocol of pure acquisition trials. We therefore take for clarity the acquisition trials of P-4 to be:

Trials 7, 8, 9 are included but set off by dotted lines because they appear to indicate a failure of strategy or memory on S-3's part.

Table 2. Acquisition Trials for P-4, s-3.

6 x 6 Task

S = {1, 2, 3, 4, 5, 6}

R = {s, t, u, v, w, x}

| i | $Q_i$ | 'ANS' | $INC_i$ |
|---|---|---|---|
| 1 | 1 - s, t, u | N | 1.0 |
| 2 | 1 - v, w | N | 1.5850 |
| 3 | 1 - x | Y | 0 |
| 4 | 2 - s, t, u | Y | .7370 |
| 5 | 2 - s, t | Y | .5850 |
| 6 | 1 - x · 2 - s | N | 1.0 |
| 7 | 2 - s,t,u | Y | 0 |
| 8 | 2 - s,t | Y | 0 |
| 9 | 2 - s | N | 0 |
| 10 | 1 - x · 2 - t | Y | 0 |
| 12 | 3 - s,u | N | 1.0 |
| 14 | 3 - v | Y | 1.0 |
| 17 | 4 - v, u | Y | 1.5850 |
| 18 | 4 - u | Y | 0 |
| 21 | 5 - w | N | 1.0 |
| 22 | 6 - w | Y | 0 |
| 23 | 1-x · 2-t · 3-v · 4-u · 6-w · 5-s | Y | 0 |

Note that trial 9 is an error according to Definition 11. Trial 23 is included but set off from other trials because it is the criterion performance.

We now turn to a trace of Goal for this task. We will use indentation to indicate changes from one level to another (i.e. from Subgoal to Search). Considerable detail is included to indicate almost every step in the acquisition process as monitored by Goal. Questions that are generated by Subgoal are indicated by an asterisk (*) and these questions and answers are compiled separately by the end for comparison with P-4.

Table 3. Trace For P-4

$S = \{1, 2, 3, 4, 5, 6\}$

$R = \{s, t, u, v, w, x\}$

$C(R) = 6$

ENTER GOAL
STORE S, R, C(R)
$j = 1$
$M(R) - \phi$
$M(S) = \phi$
$M(S) \neq S$

    ENTER SUBGOAL (1)
    $K = 1$
    $i = 1$
    $t = 3$

        ENTER SELECT $(R_{1,3}^{\cdot})$
        $R_o = R$
        $R_{1,3} = 0$
        $1 \neq 3$
        $K_1 = 1$

$K_1 < 3$

$R_{1,3} = 0 \quad \{S\} = \{S\}$

$K_1 = 2$

$K_1 < 3$

$R_{1,3} = \{S\} \cup \{t\} = \{s,t\}$

$K_1 = 3$

$K_1 < 3$

$R_{1,3} = \{s,t\} \cup \{u\}$

$K_1 = 4$

$K_1 \neq 3$

EXIT $R_{1,3} = \{s,t,u\}$

* ASK 1-S,t,u? N
$K = 2$
$i = 4$
$t = 5$

ENTER SELECT ($R_{4,5}$)

$R_o = R$

$R_{4,5} = 0$

$4 \neq 5$

$K_1 = 4$

$K_1 < 5$

$R_{4,5} = 0 \quad \{v\} - \{v\}$

$K_1 = 5$

$5_1 < 5$

$R_{4,5} = \{v\} \cup \{w\} = \{v,w\}$

$K_1 = 6$

$K_1 \neq 5$

EXIT $R_{4,5} = \{v,w\}$

* ASK 1-v,w? N
$K = 3$
$i = 6$
$t = 6$

ENTER SELECT $R_{6,6}$

$R_o = R$

$R_{6,6} = \phi$
$6 = 6$
$R_{6,6} = \{x\}$
EXIT $R_{6,6} = \{x\}$

* ASK 1-x? Y
K = 4
$C(R_{66}) = 1$
$M(R) = \phi \quad \{x\} = \{x\}$
$M(S) = \phi \quad \{1\} = \{1\}$
$R_o = \{s,t,u,v,w\}$
STORE 1-x IN STM
EXIT

j = 2
$M(X) \neq S$

ENTER SUBGOAL (2)
K = 1
i = 1
t = 3

ENTER SELECT ($R_{1,3}$)
:
EXIT $R_{1,3} = \{s,t,u\}$

* ASK 2-s,t,u? Y
K = 2
$C(R_{1,3}) \neq 1$
t = 2

ENTER SELECT ($R_{1,2}$)
:
EXIT $R_{1,2} = \{s,t\}$

* ASK 2-s,t? Y
K = 3
$C(R_{2,4}) \neq 1$
t = 1

ENTER SELECT $R_{1,1}$
$R_o = R - \{x\} = \{s,t,u,v,w\}$

$R_{1,1} = \phi$
$l = 1$

$R_{1,1} = s$
EXIT $R_{1,1}$ s

* ASK 2-s? N
K = 4
i = 2
t = 2

ENTER SELECT $R_{2,2}$
⋮
EXIT $R_{2,2} = \{t\}$

* ASK 2-t? Y
K = 5
$C(R_{2,2}) = 1$

$M(R) = \{x\} \cup \{t\}$
$M(S) = \{i\} \cup \{2\}$
$R_o = \{s, u, v, w\}$
STORE 2-5 IN STM
EXIT

j = 3
$M(S) \neq S$

ENTER SUBGOAL (3)
K = 1
i = 1
t = 2

ENTER SELECT $(R_{1,2})$
$R_{1,2} \neq \phi$
$l \neq 2$
$K_1 = 1$
$K_1 < 2$
$R_{1,2} = \phi \cup \{s\}$
$K_1 = 2$
$K_1 < 2$
$R_{1,2} = \{s\} \cup \{u\}$
$K_1 = 3$

$K_1 < 3$
EXIT $R_{1,2} = \{s,u\}$

* ASK 3-s,u?    N
$K = 2$
$i = 3$
$t = 3$

ENTER SELECT $(R_{3,3})$
$R_{3,3} = \phi$
$3 = 3$
$R_{3,3} = \{v\}$
EXIT $R_{3,3} = \{v\}$

* ASK 3-v?    Y
$K = 3$
$C(R_{3,3}) = 1$
$M(R) = \{x,t,v\}$
$M(S) = \{1,2,3\}$
$R_o = \{s,u,w\}$

$j = 4$
$M(S) \neq S$

ENTER SUBGOAL (4)
$K = 1$
$i = 1$
$t = 2$

ENTER SELECT $(R_{1,2})$
$R_{1,2} = \phi$
$1 \neq 2$
$K_1 = 1$
$K_1 < 2$
$R_{1,2} = \phi \cup \{s,u\}$
$K_1 = 2$
$K_1 < 2$
$R_{1,2} = \{s\} \cup \{u\}$
$K_1 = 3$
$K_1 < 2$
EXIT $R_{1,2} = \{s,u\}$

* ASI 4-s,u?    Y
K = 2
$C(R_{1,2}) \neq 1$
t = 2

ENTER SELECT $R_{2,2}$
$R_{2,2} = \phi$
2 = 2
$R_{2,2} = \{u\}$
EXIT $R_{2,2} = \{u\}$

* ASK 4-u?    Y
K = 3
$C(R_{2,2}) = 1$
$M(R) = \{x,t,v,u\}$
$M(S) = \{1,2,3,4\}$
$R_o = \{s,w\}$
STORE 4-u  IN STM
EXIT

j = 5
$M(S) \neq s$

ENTER SUBGOAL (5)
K = 1
i = 1
t = 1

ENTER SELECT $R_{1,1}$
$R_{1,1} = \phi$
1 = 1
$R_{1,1} = \{s\}$
EXIT $R_{1,1} = \{s\}$

* ASK 5-s?    Y
K = 2
$C(R_{1,1}) = 1$
$M(R) = \{s,t,u,v,x\}$
$M(S) = \{1,2,3,4,5\}$
$R_u = \{w\}$
STORE 5-s  IN STM
EXIT

j = 6
$M(S) \neq S$

ENTER SUBGOAL (6)
$K = 1$
$i = 1$
$t = 1$

ENTER SELECT $R_{1,1}$
$R_{1,1} = \phi$
$l = 1$
$R_{1,1} = \{w\}$
EXIT $R_{1,1}$ $\{w\}$

* ASK 6-w? Y
$K = 2$
$C(R_{1,1}) = 1$
$M(R) = \{s,t,u,v,w,x\}$
$M(S) = \{1,2,3,4,5,6\}$
$R_o = \phi$
STORE 6-w IN STM
EXIT.

$j = 7$
$M(S) = S$
EXIT

(End of Table)

Upon comparison of these questions with P-4, several differences are apparent. S-3 seems to incorporate rehearsal of 1-x in questions acquiring information about 2 (see trials 6,10). This causes a difference between P-4 and the trace of Goal. Notice that it is in this section of P-4 that the only error occurs (trial 9). Also S-3 differs from the trace at trial 17 asking 4-v,u? when the program generates 4-S,j?. Notice also that S-3's inclusion of v suggests a possible malfunction of processing. In trial 14, v has been found to match 3 and hence 4-v,u? can elicit no more information than 4-u? and

Table 4. Questions Generated by Goal.

| i | Q | ANS |
|---|---|---|
| 1. | 1 - s,t,u | N |
| 2. | 1 - v,s | N |
| 3. | 1 - x | Y |
| 4. | 2 - s,t,u | Y |
| 5. | 2 - s,t | Y |
| 6. | 2 - s | N |
| 7. | 2 - t | Y |
| 8. | 3 - s,u | N |
| 9. | 3 - v | Y |
| 10. | 4 - s,u | Y |
| 11. | 4 - u | Y |
| 12. | 5 - s | Y |
| 13. | 6 - w | Y |

hence 4-v,u? seems to be a departure from the search strategy observed in other trials. It is therefore the case that Goal fits nicely with that part of P-4 which does not contain error or anomaly.

## $\underline{\text{Subgoal}_2}$

We look next at a second strategy observed only in S-2 but which is of special interest due to its regularity. In P-10 this subject displays a strategy in which he partitions the whole response set into triples. From these triples he subtracts responses marked as matched and then asks about the remaining members of the triple. In the answers "no" he goes to the next triple. If the answer is "yes" he

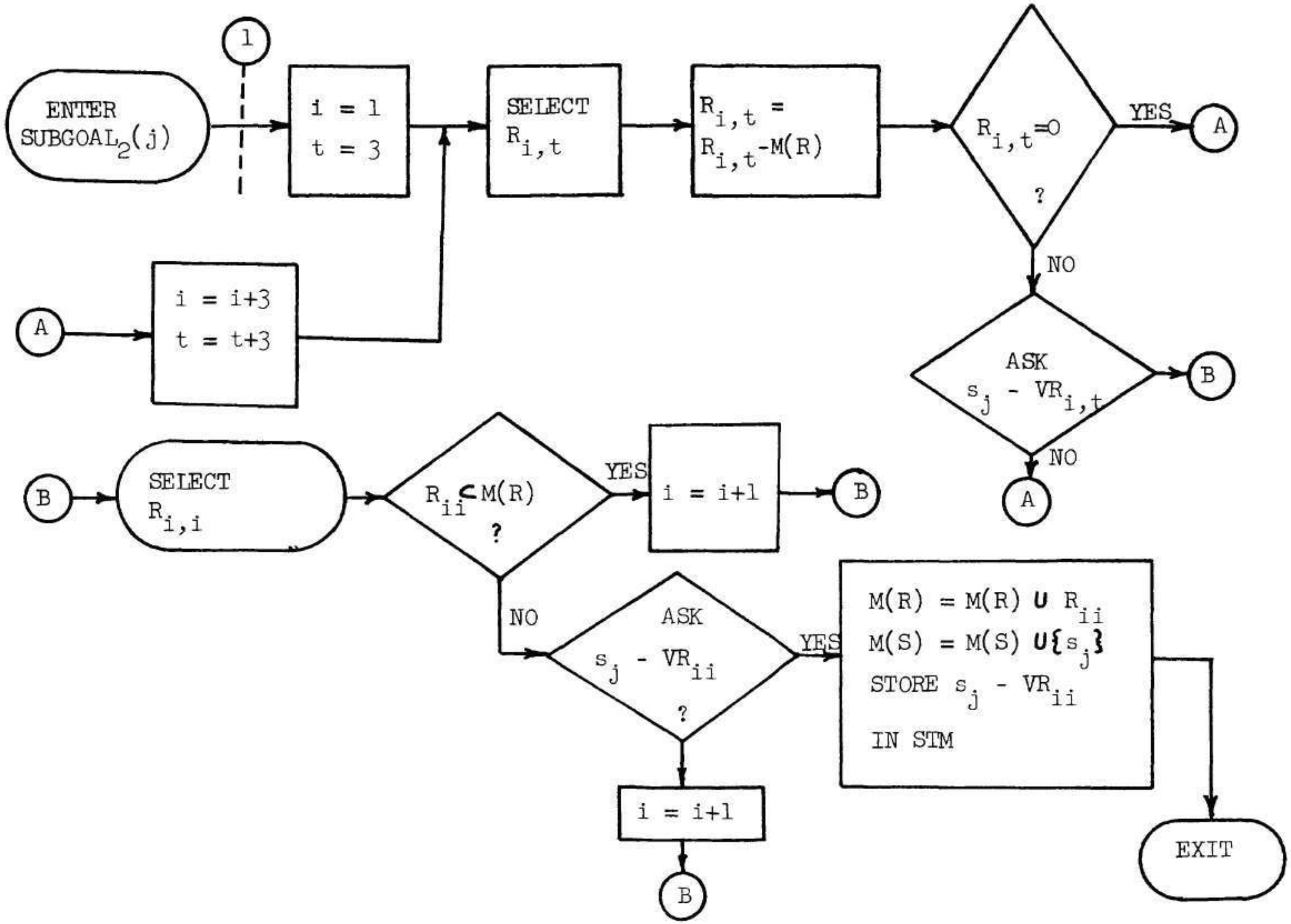


Figure 20. Subgoal$_2$ Routine.

proceeds one at a time to ask about the members of the triple until a match is found. This strategy, though very regular through trial 53, is changed abruptly at that point. We will first demonstrate a program for trials 1-46. However, dotted lines are shown across the flow chart, indicating where the program will be altered to account for the remaining trials. This $\text{Subgoal}_2$ program has one parameter, j, which is passed to it from Goal just as in the case $\text{Subgoal}_1$... The same SELECT routine is used as demonstrated above.

$\text{Subgoal}_2$ first sets an upper and lower bound which when passed to Select causes a triple of responses to be selected. This triple then has subtracted from it any marked responses. If the result is empty the next triple is selected. Otherwise a question is generated. If the jth stimulus does not match the current triple the next triple is selected. If it does match one member of this triple then up to three questions are generated to locate which member of the triple the jth stimulus matches. This is then stored along with changes in the set of marked stimuli and responses.

For comparison we will first enter the acquisition trials of P-10 to the point where S-2 changes strategy. Again, as above, a few trials are included which are by definition rehearsals but which demonstrate information which was available from inference and would otherwise not appear. The Goal, $\text{Subgoal}_2$, Select program accounts for these trials as well as the pure acquisition trials.

Table 5. Acquisition Trials for P-10, S-2, 18 x 18 Task.

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18}

R = {a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r}

| i | Qi | ANS |
|---|---|---|
| 1. | 1 - a,b,c | N |
| 2. | 1 - d,e,f | Y |
| 3. | 1 - d | N |
| 4. | 1 - e | Y |
| 6. | 2 - a,b,c | N |
| 7. | 2 - 3,f | N |
| 8. | 2 - g,h,i | N |
| 9. | 2 - j,k,l | Y |
| 10. | 2 - j | N |
| 11. | 2 - k | N |
| 12. | 2 - l | Y |
| 15. | 3 - a,b,c | N |
| 16. | 3 - d,f | N |
| 17. | 3 - g,h,i | N |
| 18. | 3 - k,m | N |
| 19. | 3 - n,o,p | N |
| 20. | 3 - q | N |
| 21. | 3 - r | N |
| 22. | 3 - a,b,c | N |
| 23. | 3 - d,f | N |
| 25. | 3 - g,h,i | N |
| 26. | 3 - j,k | Y |
| 27. | 3 - k | N |
| 28. | 3 - j | Y |
| 33. | 4 - a,b,c | N |
| 34. | 4 - d,f | N |
| 35. | 4 - g,h,i | N |

-continued-

Table 5. Acquisition Trials for P-10, S-2, 18 x 18 Task. (Continued)

| i | Qi | ANS |
|---|---|---|
| 36. | 4 - k | N |
| 37. | 4 - m,n,o | N |
| 38. | 4 - p,q,r | Y |
| 39. | 4 - p | N |
| 40. | 4 - q | Y |

Trials 18-27 are included but set apart by dotted lines. These trials include several errors (marked by!) and the whole episode seems to have been the result of faulty application of the strategy. Notice that in trial 18 a response item (j) is omitted which is not marked. Also for the next several trials the triples (k,l,m) (n,v,p) indicate that (j) has been dropped out causing the errors which occur below. The triple (k,l,m) is reduced to (k,m) as a result of (l) being marked as matched. However if (j) had been marked (in error) as matched then the next triple after (g,h,i) would have contained only (k) rather than (k,m) and the following triple would have been (m,n,o). Since this is not the case we assume that (j) is not erroneously marked but that it is omitted by an error in the selection of triples.

We now enter a trace of Goal, $\text{Subgoal}_2$, Select. Considerable detail will be shown in the body of the trace with a compilation of the generated questions being compiled at the end.

Table 6. Trace for P-10.

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

$R = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r\}$

ENTER GOAL
STORE S, R, C(R)
$j = 1$
$M(S) = \phi$
$M(R) = \phi$
$M(S) \neq S$

ENTER SUBGOAL (1)
$i = 1$
$t = 3$

ENTER SELECT $R_{1,3}$
$R_{1,3} = \phi$
$l = 3$
$k_1 = 1$
$k_1 < 3$
$R_{1,3} = \phi \quad \{a\} = \{a\}$
$k_1 = 2$
$k_1 < 3$
$R_{1,3} = \{a\} \cup \{b\} = \{a,b\}$
$k_1 = 3$
$k_1 < 3$
$R_{1,3} = \{a,b\} \cup \{c\} = \{a,b,c\}$
$k_1 = 4$
$k_1 < 3$
EXIT $R_{1,3} = \{a,b,c\}$

$R_{1,3} = \{a,b,c\} - M(R) = \{a,b,c\}$
$R_{1,3} \neq \phi$
ASK $s_1 - VR_{1,3}$ 1 - a,b,c? N
$i = 4$
$t = 6$

ENTER SELECT $R_{4,6}$

$R_{4,6} = \phi$

$4 \neq 6$

$k_1 = 4$

$k_1 < 6$

$R_{4,6} = \phi \cup \{0\}$

$k_1 = 5$

$k_1 < 6$

$R_{4,6} = \{0\} \cup \{E\}$

$k_1 = 6$

$k_1 < 6$

$R_{4,6} = \{d,e\} \cup \{f\}$

$k_1 = 1$

$k_1 < 1$

EXIT $R_{4,6} = \{d,e,f\}$

$R_{4,6} = \{d,e,f\} - M(R) = \{d,e,f\}$

$R_{4,6} \neq \phi$

ASK 1 - d,e,f? Y

ENTER SELECT $R_{4,4}$

$R_{4,4} = \phi$

$4 = 4$

$R_{4,4} = \{0\}$

EXIT $R_{4,4} = \{0\}$

$R_{4,4}$ M(R)

ASK 1 - D? N

$i = 5$

ENTER SELECT $R_{5,5}$

$R_{5,5} = \phi$

$5 = 5$

$R_{5,5} = \{e\}$

EXIT $R_{5,5} = \{e\}$

$R_{55} \not\subset M(R)$
ASK 1 - e? Y
$M(R) = \phi \cup \{e\}$
$M(S) = \phi \cup \{1\}$
STORE 1-e IN STM
EXIT

$M(S) \neq S$
$j = 2$
REVERSE

ENTER SUBGOAL (2)
$i = 1$
$t = 3$

ENTER SELECT $R_{1,3}$
⋮
EXIT $R_{4,6} = \{d,e,f\}$

$R_{4,6} = \{d,e,f\} - \{e\}$
$R_{4,6} \neq \phi$
ASK 2-d,f·? N
$i = 7$
$t = 9$

ENTER SELECT $R_{7,9}$
⋮
EXIT $R_{7,9} = \{y,h,i\}$

$R_{7,9} = \{g,h,i\} - \{e\}$
$R_{7,9} \neq \phi$
ASK 2-g,h,i? N
$i = 10$
$t = 12$

ENTER SELECT $R_{10,12}$
⋮
EXIT $R_{10,12} = \{j,k,l\}$

$R_{10,12} = \{j,k,l\} - \{e\}$
$R_{10,12} \neq \phi$
ASK 2-j,k,l? Y

ENTER SELECT $R_{10,10}$

⋮

EXIT $R_{10,10} = \{j\}$

$R_{10,10} \not\subset M(R)$
ASK 2-j? N
$i = 11$

ENTER SELECT $R_{11,11}$

⋮

EXIT $R_{11,11} = \{k\}$

$R_{11,11} \not\subset M(R)$
ASK 2-k? N
$i = 12$

ENTER SELECT $R_{12,12}$

⋮

EXIT $R_{12,12} = \{l\}$

$R_{12,12} \not\subset M(R)$
ASK 2-l? Y
$M(R) = \{e\} \cup \{l\} = \{e,l\}$
$M(S) = \{1\} \cup \{2\} = \{1,2\}$
STORE 2-l IN STM
EXIT

$j = 3$
$M(S) \neq S$

ENTER SUBGOAL (3)
$i = 1$
$t = 3$

ENTER SELECT $R_{1,3}$

⋮

EXIT $R_{1,3} = \{a,b,c\}$

$R_{1,3} = \{a,b,c\} - \{e,l\}$
$R_{1,3} \neq \phi$
ASK 3-a,b,c? N
$i = 4$
$t = 6$

ENTER SELECT $R_{4,6}$
⋮
EXIT $R_{4,6} = \{d,e,f\}$

$R_{4,6} = \{d,e,f\} - \{e,l\} = \{d,f\}$
$R_{4,6} \neq \phi$
ASK 3-d,f?   N
i = 7
t = 9

ENTER SELECT $R_{7,9}$
⋮
EXIT $R_{7,9} = \{g,h,i\}$

$R_{7,9} = \{g,h,i\} - \{e,l\}$
$R_{7,9} = \phi$
ASK 3-g,h,i?   N
i = 10
t = 12

ENTER SELECT $R_{10,12}$
⋮
EXIT $R_{10,12} = \{j,k,l\}$

$R_{10,12} = \{j,k,l\} - \{e,l\} = \{j,k\}$
$R_{10,12} \neq \phi$
ASK 3-j,k?   Y

ENTER SELECT $R_{10,10}$
⋮
EXIT $R_{10,10} = \{j\}$

$R_{10,10} \not\subset M(R)$
ASK 3-j?   Y
$M(R) = \{e,l\} \cup \{j\}$
$M(S) = \{1,2\} \cup \{3\}$
STORE 3-j IN STM
EXIT

j = 4
$M(S) \neq S$

ENTER SUBGOAL (4)
$i = 1$
$t = 3$

ENTER SELECT ($R_{1,3}$)

⋮

EXIT $R_{1,3} = \{a,b,c\}$

$R_{1,3} = \{a,b,c\} - \{e,l,j\}$
$R_{1,3} \neq \phi$
ASK 4-a,b,c? N
$i = 4$
$t = 6$

ENTER SELECT ($R_{4,6}$)

⋮

EXIT $R_{4,6} = \{d,e,f\}$

$R_{4,6} = d,e,f\} - \{e,l,j\} = \{d,f\}$
$R_{4,6} \neq \phi$
ASK 4-d,f? N
$i = 7$
$t = 9$

ENTER SELECT ($R_{7,9}$)

⋮

EXIT $R_{7,9} = \{g,h,i\}$

$R_{7,9} = \{g,h,i\} - \{e,l,j\}$
$R_{7,9} \neq \phi$
ASK 4-g,h,i? N
$i = 10$
$t = 12$

ENTER SELECT $R_{10,12}$

⋮

EXIT $R_{10,12} = \{j,k,l\}$

$R_{10,12} = \{j,k,l\} - \{e,l,j\} = \{k\}$
$R_{10,12} \neq \phi$
ASK 4-k? N
$i = 13$
$t = 15$

ENTER SELECT $R_{13,15}$

⋮

EXIT $R_{13,15} = \{m,n,o\}$

$R_{13,15} = \{m,n,o\} - \{e,l,j\}$
$R_{13,15} \neq \phi$
ASK 4-m,n,o? N
$i = 16$
$t = 18$

ENTER SELECT $(R_{16,18})$

⋮

EXIT $R_{16,18} = \{p,q,r\}$

$R_{16,18} = \{p,q,r\} - \{e,l,j\}$
$R_{16,18} \neq \phi$
ASK 4-p,q,r? Y

ENTER SELECT $(R_{16,16})$

⋮

EXIT $R_{16,16} = \{p\}$

$R_{16} \not\subset M(R)$
ASK 4-p? N
$i = 17$

ENTER SELECT $(R_{17,17})$

⋮

EXIT $R_{17,17} = \{q\}$

$R_{17,17} \not\subset M(R)$
ASK 4-q? Y
$M(R) = \{e,l,j\} \cup \{q\}$
$M(S) = \{1,2,3\} \cup \{4\}$
STORE 4-q IN STM
EXIT

END OF TABLE

Table 7. Questions generated by $Subgoal_2$.

| i | Qi | ANS |
|---|---|---|
| 1. | 1-a,b,c | N |
| 2. | 1-d,e,f | Y |
| 3. | 1-d | N |
| 4. | 1-e | Y |
| 5. | 2-a,b,c | N |
| 6. | 2-d,f | N |
| 7. | 2-g,h,i | N |
| 8. | 2-j,k,l | Y |
| 9. | 2-j | N |
| 10. | 2-k | N |
| 11. | 2-l | Y |
| 12. | 3-a,b,c | N |
| 13. | 3-d,f | N |
| 14. | 3-g,h,i | N |
| 15. | 3-j,k | Y |
| 16. | 3-j | Y |
| 17. | 4-a,b,c | N |
| 18. | 4-d,f | N |
| 19. | 4-g,h,i | N |
| 20. | 4-k | N |
| 21. | 4-m,n,o | N |
| 22. | 4-p,q,r | Y |
| 23. | 4-p | N |
| 24. | 4-q | Y |

There are very small differences between the output of the program and the acquisition trials of the protocol. One difference is observed between question 6 of the program output and S-2's trial 7

(The difference in numbering is a result of an interposed rehearsal question at trial 5). S-2 asks "does 2 match e or f?" or 2-e,f?. The program generates 2-d,f'.. This is because (e) is marked by the program as matched to (1). As expected, the output of the program differs from S-2 in the episode involving the third stimulus where S-2 has four errors. The output of the program seems to be what S would have done if he had followed the same strategy as elsewhere in the first 45 trials.

## Partition

At trial 46, S-2 begins an alteration of his apparent strategy. At this point the program described above will not account for his behavior. The changes required are not great, however, and are incorporated at another level not because the new program functions are psychologically distinct but simply because these functions are incorporated ad hoc to meet changes which are observed in the protocol. No apparent reason has been discovered why S-2 changed at this point. S-2's change in strategy is simply that he asks (after $t_{46}$) for each subsequent stimulus if it does or does not match an item in the first half of the response list. Based on the information obtained, he follows the previous strategy on either the first or the last half of the list.

The following algorithm is to be placed in $\text{Subgoal}_2$ at the location indicated by the dotted line marked 1. It accepts no parameters from the parent program and exists by changing $R_o$ (the set of available responses) to either the first or the last half of the response list.

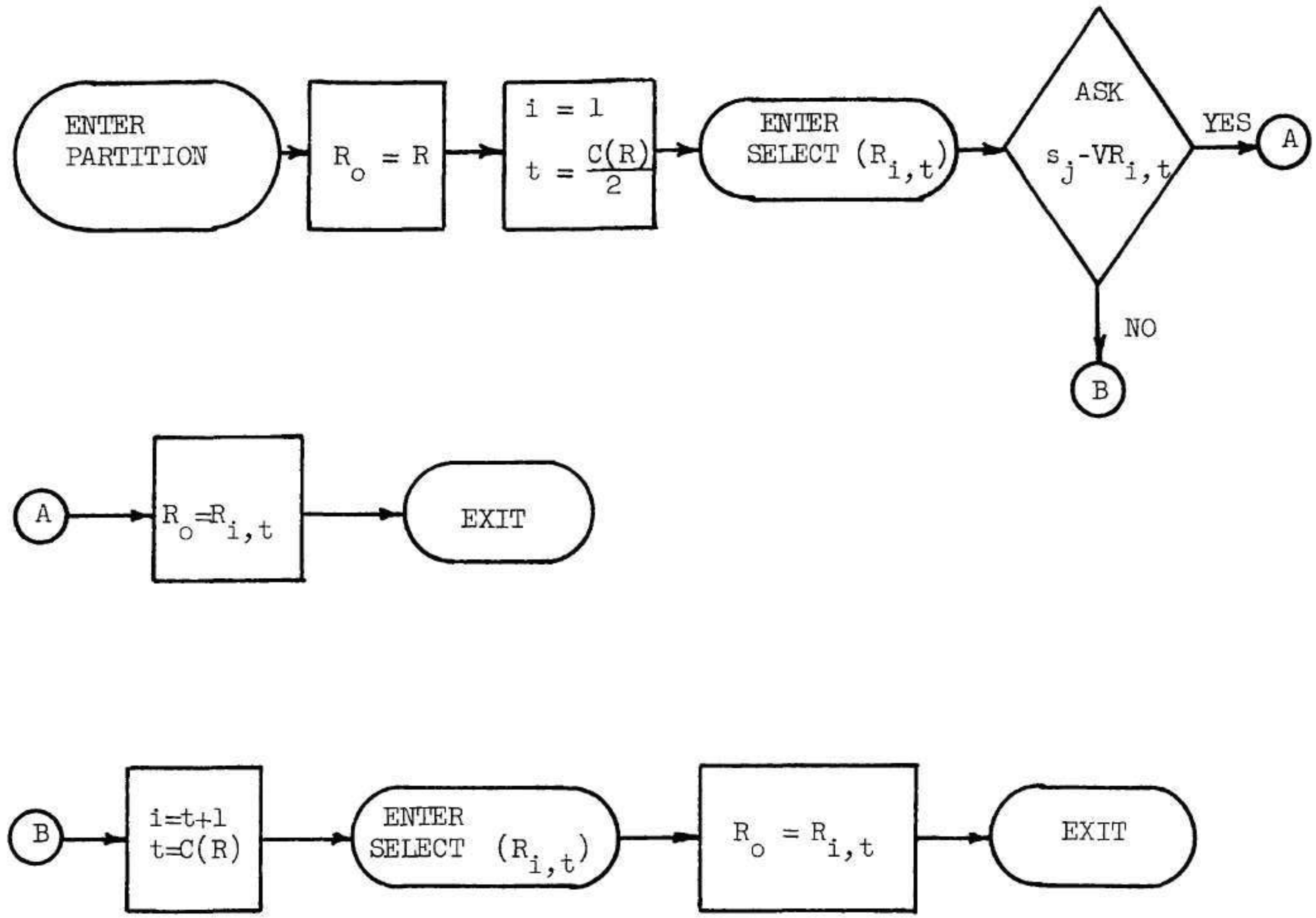


Figure 21. Partition Routine.

A trace of Goal, Subgoal, Partition, and Select should resume where the previous trace of the program without Partition terminates. This is at $j = 5$. $M(S)$ and $M(R)$ are brought over from the previous trace.

Table 8. Trace For Partition.

$j = 5$
$M(S) \neq S$

ENTER SUBGOAL (5)

ENTER PARTITION
$R_o = R$
$i = 1$
$t = 9$

ENTER SELECT
:
EXIT $R_{1,9} = \{a,b,c,d,e,f,g,h,i\}$

ASK 5-a,b,c,d,e,f,g,h,i? Y
$R = \{a,b,c,d,e,f,g,h,i\}$
EXIT

$i = 1$
$t = 3$

ENTER SELECT $(R_{1,3})$
:
EXIT $R_{1,3} = \{a,b,c\}$

$R_{1,3} = \{a,b,c\}$
$R_{1,3} = \phi$
ASK 5-a,b,c? N
$i = 4$
$t = 6$

ENTER SELECT $(R_{4,6})$
:
EXIT $R_{4,6} = \{d,e,f\}$

$R_{4,6} = \{d,f\}$
$R_{4,6} \neq \phi$
ASK 5-d,f? N
i = 7
t = 9

ENTER SELECT $(R_{7,9})$
:
EXIT $R_{7,9} = \{g,j,i\}$

$R_{7,9} = \{g,h,i\}$
$R_{7,9} = \phi$
ASK 5-g,h,i? Y

ENTER SELECT $(R_{7,7})$
:
EXIT $R_{7,7} = \{g\}$

$R_{7,7} \not\subset M(R)$
ASK 5-g? N
i = 8

ENTER SELECT $(R_{8,8})$
:
EXIT $R_{8,8} = \{h\}$

$R_{8,8} \not\subset M(R)$
ASK 5-h? N
i = 9

ENTER SELECT $(R_{9,9})$
:
EXIT $R_{9,9} = \{i\}$

$R_{9,9} \not\subset M(R)$
ASK 5-i? Y
$M(R) = \{e,l,j,q\} \cup \{i\}$
$M(S) = \{1,2,3,4\} \cup \{5\}$
STORE 5-i IN STM

$M(S) \neq S$
$j = 6$

ENTER SUBGOAL (6)

ENTER PARTITION
$R_o = R$
$i = 1$
$t = 9$

ENTER SELECT $(R_{1,9})$
:
EXIT $R_{1,9} = \{a,b,c,d,e,f,g,h,i\}$

ASK 6-a,b,c,d,e,f,g,h,i? Y
$R_9 = \{a,b,c,d,e,f,g,h,i\}$
EXIT

$i = 1$
$t = 3$

ENTER SELECT $(R_{1,3})$
:
EXIT $R_{1,3} = \{a,b,c\}$

$R_{1,3} = \{a,b,c\}$
$R_{1,3} \neq \phi$
ASK 6-a,b,c? Y

ENTER SELECT $(R_{1,1})$
:
EXIT $R_{1,1} = \{a\}$

$R_{1,1}$ $M(R)$
ASK 6-a? Y
$M(R) = \{e,l,j,q,r\} \cup \{a\}$
$M(S) = \{1,2,3,4,5\} \cup \{6\}$
STORE 6-a IN STM

End of Table

The program Partition has been demonstrated along with Goal, $\text{Subgoal}_2$, Select so we will at this point dispense with the detailed trace and only provide the output for the remainder of the interactions along with those generated above. These outputs are given in Table 9.

Table 9. Outputs of Partition.

| | | |
|---|---|---|
| 25 | 5-a,b,c,d,e,f,g,h,i | Y |
| 26 | 5-a,b,c | N |
| 27 | 5-d,f | N |
| 28 | 5-g,h,i | Y |
| 29 | 5-g | N |
| 30 | 5-h | N |
| 31 | 5-i | Y |
| 32 | 6-a,b,c,d,e,f,g,h,i | Y |
| 33 | 6-a,b,c | Y |
| 34 | 6-a | Y |
| 35 | 7-a,b,c,d,e,f,g,h,i | N |
| 36 | 7-j,k,l,m,n | Y |
| 37 | 7-k | N |
| 38 | 7-m,n | Y |
| 39 | 7-m | N |
| 40 | 7-n | Y |
| 41 | 8-a,b,c,d,e,f,g,h,i | Y |
| 42 | 8-b,c | Y |
| 43 | 8-b | N |
| 44 | 8-c | Y |
| 45 | 9-a,b,c,d,e,f,g,h,i | Y |
| 46 | 9-b | N |
| 47 | 9-d,f | Y |
| 48 | 9-d | N |

| | | |
|---|---|---|
| 49 | 9-f | Y |
| 50 | 10-a,b,c,d,e,f,g,h,i | N |
| 51 | 10-j,k,l,m,n | N |
| 52 | 10-o,p | Y |
| 53 | 10-o | Y |
| 54 | 11-a,b,c,d,e,f,g,h,i | N |
| 55 | 11-j,k,l,m,n | Y |
| 56 | 11-k | Y |
| 57 | 12-a,b,c,d,e,f,g,h,i | Y |
| 58 | 12-b | N |
| 59 | 12-d | Y |
| 60 | 13-a,b,c,d,e,f,g,h,i | N |
| 61 | 13-j,k,l,m,n | Y |
| 62 | 13-k | N |
| 64 | 13-m | Y |
| 65 | 14-a,b,c,d,e,f,g,h,i | N |
| 66 | 14-j,k,l,m,n | N |
| 67 | 14-p | N |
| 68 | 14-r | Y |
| 69 | 15-a,b,c,d,e,f,g,h,i | Y |
| 70 | 15-b | Y |
| 71 | 16-a,b,c,d,e,f,g,h,i | N |
| 73 | 16-j,k,l,m,n | N |
| 74 | 16-p | Y |
| 75 | 17-a,b,c,d,e,f,g,h,i | Y |
| 76 | 17-g,h | Y |
| 77 | 17-g | Y |
| 79 | 18-a,b,c,d,e,f,g,h,i | Y |
| 80 | 18-i | Y |

end of table

The remainder of the protocol (P-10) with which this is to be compared will not be reproduced here but can be found in Appendix I. The fit of the questions generated to the acquisition trials of P-10 is quite close. There are a few anomalies, however. For example, in questions 28-31 of the program of S-2's strategy, the line of questions about stimulus (5) is different from the line of questions in the protocol:

| PROGRAM | PROTOCOL |
|---|---|
| 5-g,h,i? | 5-g? |
| 5-g? | 5-h? |
| 5-h? | 5-i? |
| 5-i? | |

The reason is that after question 27 the inference could be made that 5-g,h,i, S-2 makes this inference, but this inference is not reflected in the program. This is the case because S-2 does not elsewhere systematically make such inferences. A similar case occurs at questions 38 and following.

| PROGRAM | PROTOCOL |
|---|---|
| 7-m,n? | 7-m? |
| 7-m? | 7-n? |
| 7-n? | |

and at question 52.

| PROGRAM | PROTOCOL |
|---|---|
| 10-o,p? | 10-o? |
| 10-o? | |

However, other anomalies can be explained by pointing to errors made by S-2. For example, at question 47.

| PROGRAM | PROTOCOL |
|---|---|
| 9-d,f? | 9-d? |
| 9-d? | 9-e? |
| 9-f? | 9-f? |

There is here an apparent case of S-2's having failed to recall (e) as being marked as matched (to (7)) in the question 9-e?. This is answered "no" and is an error by Definition 11. It would appear that S-2 fails to recover from this error for several trials as can be seen at question 58 of the program output.

| | PROGRAM | PROTOCOL |
|---|---|---|
| 58 | 12-b? | 12-e? |
| 59 | 12-d? | 12-d? |

Again an error is produced when a question, 12-e?, is asked, since (e) is marked as matched. Any other disagreements between protocol and program are minor instances of S-2 making inferences that are not regularly made and hence cannot be incorporated into the program.

# CHAPTER IV

## ANALYSIS OF EXPECTED NUMBER OF QUESTIONS

It is possible to analyze the expected number of acquisition questions which will result from the various strategies modeled above. The expected number of questions can then be compared with the actual number observed in the protocols which are modeled.

The most interesting of the strategies is the binary search because it is an approximation to a known optimal search strategy. Zimmerman [1959] has given a strategy that minimizes the expected number of yes-no questions required to isolate a prespecified object (whose identity is unknown to the subject) from a set of objects. The strategy involves using "yes or no" questions to divide the set of objects into two groups in a continuing process until the object is determined. Zimmerman observes however that what seems to be a process of dividing a set is really a process of combining objects into new objects. Thus, to start with the entire set to be searched and proceed by partitioning until only one object is in each cell, is equivalent to proceeding in the precisely reverse manner. One can just as well start with the several isolated objects and indicate a series of repeated combinations, the first of which corresponds to the last of the divisions, and the final one to the first division. It is then proven that the optimal procedure of combination is, at every stage, to combine the two objects that have the two smallest probabilities.

It is interesting that this question strategy is exactly the same as a coding procedure due to Huffman [1952], that was originally developed in the context of statistical information theory. Under the assumption that the messages to be encoded are selected independently according to an arbitrary probability distribution, Huffman's coding procedure leads to an instantaneous code with the smallest possible average code-word length; e.g., see Ash [1965].

Another coding procedure, due to Fano [1961], is, when interpreted as a series of "yes or no" questions, the alter ego of the binary search strategy followed by several S's. The Fano code is constructed by partitioning the ensemble into equiprobable, or as nearly as possible equiprobable, groups and subgroups. Each partitioning is assigned an additional 0.1 digit. The groups and subgroups formed by successive divisions are indicated by the digits of the code words. This is just the same as an S partitioning the available matches to a particular s and identifying the partitions with the resulting sequence of "yes or no" responses elicited. The "0" of a Fano Code is interpreted as "yes" and the "1" as "no". The result is code words which represent sequences of questions constructed in a manner identical to binary search. We enter an example of a search through seven equiprobable responses by binary search using the Fano diagram.

This procedure is known to not always produce the shortest possible average word length in cases where the group divisions cannot be made equiprobable. Discussed below is the fact that for any code, average code word length $\bar{e}$ is always greater than or equal to the entropy H of the ensemble. Therefore the approximation to optimality of a code can be measured by its efficiency $E = \frac{H}{\bar{e}}$ . Often the Fano

| $r_i$ | Code Words |
|---|---|
| $r_1$ | 000 |
| $r_2$ | 001 |
| $r_3$ | 010 |
| $r_4$ | 011 |
| $r_5$ | 100 |
| $r_6$ | 101 |
| $r_7$ | 11 |

$\bar{e} = \frac{20}{7}$

3rd div.

2nd div.

1st div.

Figure 22. Fano Code Words for Seven Equiprobable Responses.

procedure (or the binary search) produces code (question sequences) of high efficiency and is therefore a close approximation to optimality. In case that the efficiency is not high, block coding procedures can increase the efficiency; e.g., see Hyvärinen [1968] pp. 44-45. However, since there is no observed analogy to block coding in subject's search strategies, this is of no interest here.

In addition to the fact that the binary search procedure is approximately optimal, we have in statistical information theory the mechanisms for setting bounds on the expected number of questions that S requires to obtain all components of an N×N rule.

For an N×N experiment there are N possible matches for the first stimulus item for which a match is found (call this $s_1$): N-1 for the second ($s_2$), etc. So there are N! possible N-component rules which S must distinguish by his questions. For the ith component discovered in the rule there are (N-i+1)! possible N-component rules.

The minimum average number of questions S requires to discover the N components of the rule can be found by application of a corollary of the noiseless coding theorem from statistical information theory; e.g., Ash [1965] pp. 36-40. This theorem states that given a random variable X with uncertainty H(X), there exists a base D instantaneous code for X whose average code-word length $\bar{e}$ satisfies

$$\frac{H\ (X)}{\mathrm{LOG}_2\ D} \leq \bar{e} < \frac{H\ (X)}{\mathrm{LOG}_2\ D} + 1 \tag{1}$$

where

$$H(X) = -\ \Sigma\ p_i \mathrm{LOG}_2 p_i\ .$$

In order to apply this theorem to the circumstances at hand certain assumptions must be made which appear to be valid. These assumptions have to do with the relation between instantaneous codes (codes in which no code word is a prefix of another code word) and sequences of "yes or no" responses to S's questions. Ash observes without proof that '...any instantaneous binary code corresponds to a sequence of 'yes or no' questions...". He also observes that based on this assumption the uncertainty H(X) can be thought of as the minimum average number of "yes or no" questions required to fix an observation of X. Simple examples seem to support Ash's observations; however, in order for the following observations to carry maximum weight, this assumption would require more careful examination.

Based upon the above assumptions we may proceed to observe that if D=2, (1) becomes

$$H(X) \leq \bar{e} < H(X) + 1 \qquad (2)$$

This is to say that the minimum average number of "yes or no" questions required to fix one observation of X is between H(X) and H(X) + 1. Since there are N! possible matching rules for an N x N experiment, each with probability $\frac{1}{N!}$ (by assumption), equation (2) implies

$$\mathrm{LOG}_2 N! \leq \bar{e} < \mathrm{LOG}_2 N! + 1 \,. \qquad (3)$$

This means, for example, that the expected number of questions for the 18 x 18 tasks in P-10 and P-12 are bounded below by 52.5 and there is a question strategy with an average between 52.5 and 53.5

which will successfully isolate the rule. S-6 performed the acquisition of the rule in 55 acquisition trials by a nearly pure binary search while S-2 used 59 acquisition trials in what we have called the triples strategy represented in $\text{Subgoal}_2$. It is clear that this strategy is farther away from a pure binary search than the strategy followed by S-6.

For a 14 x 14 task the expected number of acquisition questions is bounded below by 36.3 questions and there exists a strategy with an average between 36.3 and 37.3 questions which will isolate the rule.

S-3, following a binary search as characterized by $\text{Subgoal}_1$, requires 38 acquisition trials. An interesting comparison for this size task is P-9 by S-5 who followed a binary search strategy (to some extent) only after trial 22. Up to that point a linear search was followed. S-5 uses 52 acquisition trials, even though we know a strategy exists with fewer than 50.3 questions. In order to really decide how well S-5 has done with his mixed strategy we need to know the average number of questions required to isolate the rule using a pure linear search.

A "linear search" will mean here that S asks in order "does $s_i$ match $r_1$?", "does s match $r_2$?", etc., until a "yes" answer is obtained. We can calculate how many questions on the average are in the sequence which fixes the rule component for $s_i$. Let the rule be an N x N rule. For $s_1$ there are n equiprobable matches. For each $r_i$ the number of questions required to discover that $s_1$ matched $r_i$ is i.

| RESPONSE | PROBABILITY | NUMBER OF QUESTIONS |
|---|---|---|
| $r_1$ | 1/N | 1 |
| $r_2$ | 1/N | 2 |
| $r_3$ | 1/N | 3 |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $r_{N-1}$ | 1/N | N-1 |
| $r_N$ | 1/N | N-1 |

The last possibility is marked by a "no" answer to the n-1th question. The expected number of questions, $\bar{e}_1$, can be found as follows

$$\bar{e}_1 = \frac{1}{N}\left[\left(\sum_{i=1}^{N-1} i\right) + N-1\right]$$

or

$$\bar{e}_1 = \frac{1}{N}\left[\left(\sum_{i=1}^{N} i\right) - 1\right]$$

from the formula $\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$ we obtain

$$\bar{e}_1 = \frac{N+1}{2} - \frac{1}{N}$$

Since there are N-1 possible matches for $s_2$ we have:

$$\bar{e}_2 = \frac{N}{2} - \frac{1}{N-1}$$

For the ith stimulus

$$\bar{e} = \frac{(N - i + 1) + 1}{2} - \frac{1}{N - i + 1}$$

If $\bar{e}_i$ is the expected number of questions for all N components of the rule then

$$\bar{e} = \sum_{i=1}^{N} \bar{e}_i$$

$$\bar{e} = \sum_{i=1}^{N} \left(\frac{(N - i + 1) + 1}{2} - \frac{1}{N - i + 1}\right)$$

$$= \sum_{i=1}^{N} \frac{(N - i + 1) + 1}{2} - \sum_{i=1}^{N} \frac{1}{N - i + 1}$$

$$= \frac{1}{2} \sum_{i=1}^{N} (N - i + 1) + \frac{N}{2} - \sum_{i=1}^{N} \frac{1}{N - i + 1}$$

$$= \frac{1}{2} \sum_{i=1}^{N} i + \frac{N}{2} - \sum_{i=1}^{N} \frac{1}{i}$$

$$= \frac{1}{2} \left(\frac{N(N + 1)}{2}\right) + \frac{N}{2} - \sum_{i=1}^{N} \frac{1}{i}$$

For a 14 x 14 task this produces $\bar{e} = 56.25$ compared to 52 questions for S-5 in P-9.

It is also possible to calculate the expected number of questions for the "triples" strategy. This strategy can be viewed as dividing the available responses into triple units and then doing a linear search over these. When the correct triple is found a linear search is used to isolate the correct response in that triple. No neat formula was found for this prediction but values for n=5 and n=18 were calculated by enumeration; these are 7.3 and 57.9 questions respectively. These predictions fit very nicely with the actual values of 8 and 59 questions. Table 10 displays a comparison of all protocols with theoretical results. Included are the actual number of acquisition trials for each protocol, the upper and lower bounds for the existence of a strategy and the average number of questions which would result from a linear search. It is interesting that the only protocols which fail to fall between the upper and lower bounds for a minimum strategy are P-9 and P-11. P-11 is a case in which a subject failed to discover the rule at all. P-9 represents a case in which a subject followed a linear search temporarily. P-2 and P-10 represents cases in which S-2 followed the "triples" strategy causing the actual number of acquisition trials to exceed the theoretical minimum average by several trials. In all other cases a binary search strategy was followed causing the actual information acquisition trials to fall very close to the theoretically minimum average.

Table 10. Expected Number of Questions.

| PROTOCOL | S | NUMBER ITEMS | NUMBER OF ACQUISITION TRIALS | A STRATEGY EXISTS BETWEEN THESE AVERAGES ----&---- | | AVERAGE FOR LINEAR SEARCH | TRIPLES |
|---|---|---|---|---|---|---|---|
| 1a | 1 | 5 | 6 | 6.9 | 7.9 | 7.72 | |
| 1 | 1 | 5 | 7 | 6.9 | 7.9 | 7.72 | |
| 2 | 2 | 5 | 8 | 6.9 | 7.9 | 7.72 | 7.3 |
| 3 | 3 | 5 | 7 | 6.9 | 7.9 | 7.72 | |
| 4 | 3 | 6 | 9 | 9.5 | 10.5 | 11.05 | |
| 5 | 3 | 7 | 12 | 12.3 | 13.3 | 14.90 | |
| 6 | 3 | 9 | 18 | 18.5 | 19.5 | 19.67 | |
| 7 | 3 | 11 | 26 | 25.2 | 26.2 | 35.48 | |
| 7a | 7 | 12 | 28 | 28.8 | 29.8 | 41.82 | |
| 8 | 3 | 14 | 38 | 36.4 | 37.4 | 56.25 | |
| 8a | 1 | 14 | 34 | 36.4 | 37.4 | 56.25 | |
| 9 | 5 | 14 | 52 | 36.4 | 37.4 | 56.25 | |
| 10 | 2 | 18 | 59 | 52.5 | 53.5 | 91.00 | 57.9 |
| 11 | 4 | 18 | Failed | 52.5 | 53.5 | 91.00 | |
| 12 | 6 | 18 | 55 | 52.5 | 53.5 | 91.00 | |

# CHAPTER V

## A MODEL OF REHEARSAL

In the chapters above we have demonstrated that S's can produce error free performances over trials in conversational tasks and have demonstrated an analysis of strategies which for those S's considered will account for the number of acquisition trials and predict their nature. Even though these error free performances are not observed in all cases, still the incidence is high enough to require that we attempt to account for how these S's can accommodate their constraints of memory to the task in such a way that error-free or nearly error-free performance is possible.

In conversational trials, S has control over several variables to which he has no access in fixed trial procedures. Some of these are rate of presentation, order of presentation, rate of rehearsal and order of rehearsed items. Glanzer and Cunitz [1966] have shown that presentation rate has no effect on the proportion of recent items recalled, yet does affect the early items. The fact that presentation rate affects early items and not items in short term memory suggests the possibility that longer inter-item periods allows implicit rehearsal and consequently increases the probability that the item is in longer term stores. This possibility is one which enters into the construction of a model of the interaction of short and long term stores by Waugh and Norman [1965] and also seems to be a factor which should be considered in the construction of an explanation of error-free performance. If it is rehearsal that increases the probability that an item is in longer term stores then rehearsal must be included in an explanation of error-free performance or any performance with high probability of recall. It is clear that if S performs with minimal error then some precedure must be available to him which allows him to decide when and in what order items must be rehearsed in order to minimize loss. This method of deciding what to rehearse and when to rehearse should take the order of presentation or discovery into account.

General consideration of error-free performance causes us to postulate that S has a "model" of his own performance. S somehow is able to "tell how he is doing" relative to some learned item. Introspective data from S's indicate that they know whether an item is firm or if they might lose it if they proceed without rehearsing. In order for S to avoid loss he must in some manner (whether consciously or not) monitor the status of items, decide whether each item can be left safely while acquiring new information, and order those items which must be rehearsed. Let us suppose then that such a model of rehearsal exists and that it is an adequate basis for such decisions. The fact that S's have been observed in error free performance indicates that they have access to such a model and that this access is utilized in forming their rehearse/acquire strategy.

What we seek, then, is a model which, based on the position or order of acquisition of an item and the rehearsal of that item, produces an estimate of the "strength" of that item. Let us assume that this "strength" is probability of recall and that this probability is available to S as a subjective probability. We do not assume that S can accurately externalize these subjective probabilities but only that they are available to him and that he can base decisions on them. Many S's fail to perform error free or even nearly error free. These S's may not have such a model, their subjective probabilities may not be as accurate as in other S's or they simply may not use them in an adequate way in framing decisions.

The assumption that S's can utilize subjective probabilities in a reliable way is not without precedent. The work on vigilance utilizing decision theory is an example of a theoretical study involving subjective probability in a way that is nicely validated empirically. (see Watson, et. al. [1964]).

The basic notion of S's model of his own performance is that he can monitor the probability of recall of an item of earlier presentation and that when this probability falls on or below a predetermined threshold, he will rehearse that item. The rehearsal of the item, in the form of a question about the correctness of his rehearsal of that item serves to renew or increase the probability that the item can be correctly recalled again. Rehearsal considered in this way is in accord with Definition 9, Chapter I. S's were asked to cooperate in the verbalizing of all rehearsals in the form of questions. The ease with which S's seemed to do this suggests that treating rehearsal in this way is possible. Certain "quiet" intervals in which S asked no questions indicates that this treatment of rehearsal is not absolutely adequate and that introspective rehearsals without feedback do occur.

It is felt, however, that this treatment of rehearsal as though it were exclusively externalized and with feedback introduces no grave difficulties in a first pass theory. The assumption of the model that S uses knowledge of results of rehearsals in making decisions about rehearse/acquire choices suggests that S will use opportunities to obtain knowledge of results if they are available. Other studies which involve subjective probabilities in models of performance have shown the value to S of knowledge of results. In signal detection or vigilance studies, for example, knowledge of results have had a significant effect on performance (See McCormack [1962]). For these reasons we incorporate rehearsal with feedback and omit from consideration the effects of covert rehearsal.

Before explicitly constructing a model of rehearsal, a decision must be made concerning what causes a decrement in probability of recall. The controversy over spontaneous decay and interference as explanations has not been clearly resolved.

The choice is not as critical here as it might seem. We do not attempt to comment on memory itself but only on how S's control of variables affecting memory can produce error free performance. If items or trials were to occur with regularity in conversational experiments, then time rate of change could be used as a parameter in the model even if it were known that interfering items were the effective variable. S's do fall into a cadence as they elicit information; however, this cadence is not absolutely regular. Also there is no record of timing information in the protocols in Appendix I. Therefore this argument in favor of flexibility is not binding.

In an important paper on primary memory, Waugh and Normal [1965] have based a model of memory on interference of intervening items. We wish to make use of this model in explicating rehearsal and we therefore take note of their position on the decay/interference issue. These authors constructed a probe digit experiment which they argued utilized only primary memory and in which items were presented at different rates. They found that the effect of rate of presentation was very small compared to the effect of serial position. This caused them to conclude that "the main source of forgetting in our experiment was interference." This conclusion has been questioned by Broadbent [1971]. He refers to an unpublished lecture by Shallice in which the possibility is raised of two functions both affected by time being at play in The Waugh-Normal experiment. He suggests that a slow rate makes perception of the items easy and also makes forgetting easy. Hence, Broadbent argues that the average performance at a slow rate is not worse than that at a high rate.

We simply avoid the controversy by assuming that the evidence is not sufficiently strong against Waugh and Norman to prevent our accepting their assumptions in order to utilize their theory.

We obtain from Waugh and Norman the relationship between the number of interfering items and the probability of recall from primary or short term memory which is a slightly Ogive shaped curve. It can be shown that based upon this recall function, primary memory alone cannot account for a rehearsal strategy allowing error free performance. Let us assume that probability of recall decreases with intervening items according to this curve and that S's introspective model

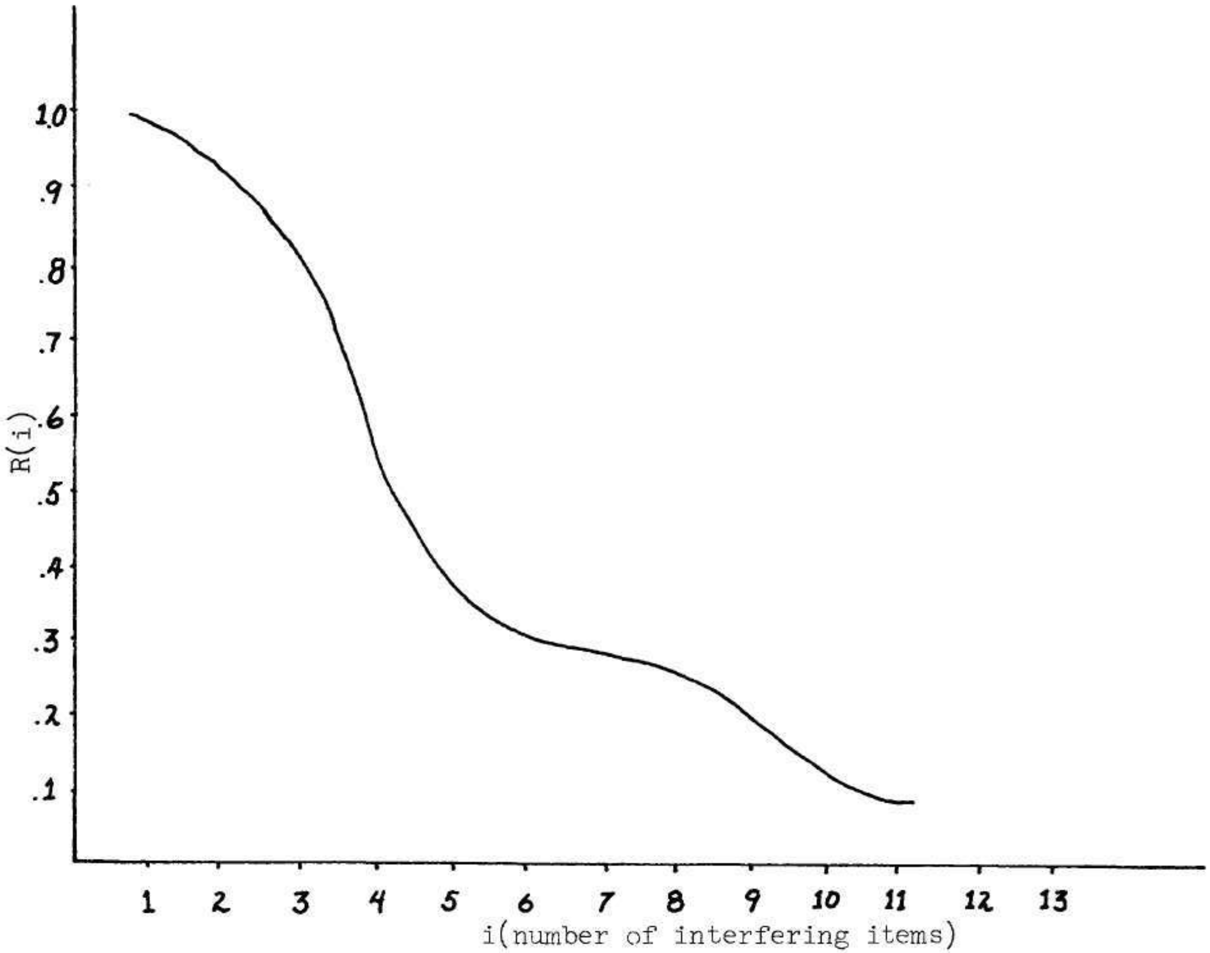


Figure 23. Probability of Recall from Primary Memory.

of his own performance involves a subjective probability curve which corresponds to it. We also assume that with each rehearsal of an item the count of intervening items is reset to zero. Let us also suppose that S sets a confidence threshold for the probability of recall at $R\ (C) \leq v$ and that if probability of recall $R(j)$ is greater than $v$ after j intervening items the item will not be rehearsed. Suppose S discovers the response which matches a particular stimulus (an item). Let this item be item 1. Then the probability of recall of item 1 is $R_1(0)$ since there are no intervening items. Let S discover item 2 which serves as an intervening item for 1. Then we have $R_2(0)$ as the probability of recall of item 2 but the probability of recall of item 1 is decremented to $R_1(1)$. The process of acquisition of new items may occur until $R_1(C) \leq v$ is encountered. At this point we have probabilities of recall of c items:

$$R_c(0) > v$$

$$R_{c-1}(1) > v$$

$$\vdots$$

$$R_2(c-1) > v$$

$$R_1(c) \leq v$$

which requires that item 1 be rehearsed. The rehearsal resets the intervening item count to zero for item 1. We shall indicate this as $R_{1,1}(0)$ with the second subscript on R indicating the number of times item 1 (the first subscript) has been rehearsed.

We assume that rehearsal of item 1 constitutes an intervening item for other items. Therefore after this rehearsal we have:

$$R_{1,1}(0) > V$$

$$R_c(1) > V$$

$$R_{c-1}(2) > V$$

$$\vdots$$

$$R_2(c) \leq V$$

and the second item must now be rehearsed. This rehearsal in similar manner causes the third item to be rehearsed. This rehearsal sequence would continue until c rehearsals would produce $R_{1,1}(c) \leq V$ causing the first item to require rehearsal. Therefore, if a model of rehearsal is based on one store of memory alone, for example on primary or short term memory, that model will predict endless loops of rehearsals after a fixed number of acquisitions. As a result we assert that any model of rehearsal which is based on the assumptions described above must in some way involve a probability of recall function which decrements less rapidly after rehearsals than it does before initial rehearsals. Otherwise the trapping state as in the above example will occur. If each rehearsal causes the function to decrement more slowly, then more and more items can be acquired or rehearsed between subsequent rehearsals of the same item.

The effect of rehearsal on performance that we hope to account for is not too different from the Hebb effect (Hebb [1961]). It was shown in an experiment investigating the physiology of memory that when many lists of digits were presented to S for immediate recall, if 0 list was repeated the probability of recall was greater the second time than it had been after the first presentation. Since several lists had been presented between the first and second presentations of the list, either spontaneous decay or interference theories of primary memory would predict no significant improvement for the second presentation. What seemed to be happening was that repetition caused transfer to longer term stores in memory, hence increasing the probability of recall.

Waugh and Norman [1965] have presented a model of recall from memory which postulates that the probability of recall from primary or short term memory and the probability of recall from secondary or long term memory are independent and thus the probability of recall is:

$$R(i) = P(i) + S(i) - P(i)\,S(i)$$

where i is the number of intervening items. This model is useful in the case where rehearsal is controlled so that the number of intervening items is the only variable. In a case in which rehearsal is permitted we also expect from the Hebb effect that the probability of recall will depend on the number of rehearsals. Waugh and Norman argue that without rehearsal $S(i)$ (the probability of recall from secondary memory) is a constant (does not vary with c) for a particular task. We suppose that this is the case and that $P(i)$ (probability of recall from primary memory) is constant with respect to the number of rehearsals. It seems to be the case that primary memory is restored with each rehearsal, that is i is reset to zero, but that the number of rehearsals does not affect the probability of recall from primary memory for a fixed value of i.

Let n be the number of rehearsals and i the number of interfering items since last rehearsal. The probability of recall from primary memory, secondary memory, or both becomes a series of partial functions as follows:

for all $n \geq 0$

$$\begin{cases} R^n(n,i) = P(i) + S(n) - P(i)\,S(n) \\ R^n(n+1,i) = R^{n+1}(n+1,0) \end{cases}$$

Thus for $n = 0$ and $i = 1,2,3,\ldots$ the model is identical to the Waugh-Norman model. i.e., $R^o(0,i) = P(i)$. However at $n = 1$, $i = 3$ (for example) we have:

$$R^o(1,3) = R^1(1,0) = P(0) + S(1) - P(0)\,S(1)$$

then as i has been reset to 0 we proceed with $n = 1$, $i = 1,2,3, \ldots$ as:

$$R^1(1,1),\ R^1(1,2),\ R^1(1,3),\ldots$$

again until n is incremented. We have chosen $n \geq 0$ rather than $n > 0$ in the definition of the model so that $R^o(0,i)$ will be $P(i)$ of the Waugh-Norman model. This is the case only if $S(0) = 0$, however. The nature of $S(N)$ is an important question. It seems reasonable that if an item has been presented one time that this first presentation is the first rehearsal in the sense that the first presentation should have the same effect on retention in secondary memory as each subsequent presentation. Therefore, the probability of recall after the first presentation of an item and prior to the next item should be $R^1(1,0) \cdot S(n)$ can be approximated from existing data. To do this we can transform the model to:

$$S(n) = \frac{R^n(n,i) - P(i)}{1 - P(i)}$$

Fairly reliable data can be found for $P(i)$. For example, the Waugh-Norman model was constructed to correct retention estimates for S to obtain a relatively pure estimate of $P(i)$. Therefore, if data can be found which record probability of recall accounting both for number of rehearsals and number of intervening items an estimate of $S(n)$ can be constructed.

The paper by Hebb mentioned above might provide an early approximation to $S(n)$. Subjects were given series of digits to repeat. However, every third trial the same series was presented. This procedure allowed Hebb to examine the effects of repeated presentation. Since each series of nine digits was presented and immediately recalled, the number of intervening items for each item is eight. This allows correcting Hebb's data for primary memory using the Waugh-Norman data. This procedure then produces:

$$S = \{(1,.32),(2,.52),(3,.69),(4,.58),(5,.58),(6,.59),(7,.78),(8,.80)\ldots\}$$

The lack of regularity of this data precludes its use.

There is strong reason to prefer a learning curve asymptoting to 1.0 for $S(n)$. The Hebb data in a general way suggest a learning curve. Probability of correct recall should be an increasing function of rehearsals in order to allow increasing intervals for additional acquisitions between rehearsals. The intercept should be zero since there should be zero probability of recovery of an unpresented item from secondary memory. Clearly probability of recall must not exceed 1.0. Our approximation of $S(n)$ must asymptote to 1.0 in order to explain performances of several subjects with perfect 24 hour recall.

An experiment can be framed in the conversational methodolgy to elicit data for an approximation of $S(n)$ for groups of S's. This would require the use of interpolated tests among conversational trials which would provide an estimate of $R^n(N,i)$. These interpolated tests would necessarily follow an item for some constant value of $i_c$ and all values of N up to 15 or so. This measure of $R^n(N,i_c)$ could then be corrected for $P(i_c)$ from Waugh-Norman data. This experiment has not been done. It is claimed that the model of rehearsal is highly tentative and that considerable future refinement is needed. A discussion of the necessary refinements will be found in Chapter VI. Since the model is tentative and requires additional refinement it is felt that it is sufficient to simply choose a learning curve of unspecified parameters for $S(n)$. The equation $S(n) = M(1 - e^{-kn})$ was chosen. In this equation M is a constant of asymptotic maximum value for $S(n)$ and must be 1.0 in order to account for instances of 100 per cent 24 hour retention. The constant K is a "rate of learning" constant and is unspecified. For a particular value of K and P(i) from Waugh and Norman's data the model of rehearsal provides a family of curves as in Figure 24.

The value of V used by a particular S can be approximately measured from a protocol of his performance. The location of the first rehearsal of the first item provides a value $R^1(1,i)$ which tells us at what probability of correct recall S decides to rehearse rather than acquire new information. For a typical value of K this provides the following values.

Table 11. Values of V.

| Rehearsal After i Items | V ≈ |
|---|---|
| 1 | .9 |
| 3 | .8 |
| 5 | .7 |

For every item (s-r pair) that S discovers with his questions, there is a unique count of N and i. The number of rehearsals is the number of times a particular item has been rehearsed. The variable i is the number of intervening items for a particular item. Therefore, as each new item is discovered, a new pair of counters for its i and n are created. The whole array of these counters for each item and their associated probabilities of recall $R^n(N,i)$ can be used to predict when rehearsals will occur and when acquisitions will occur in a protocol. The criterion performance for an m item task would be the

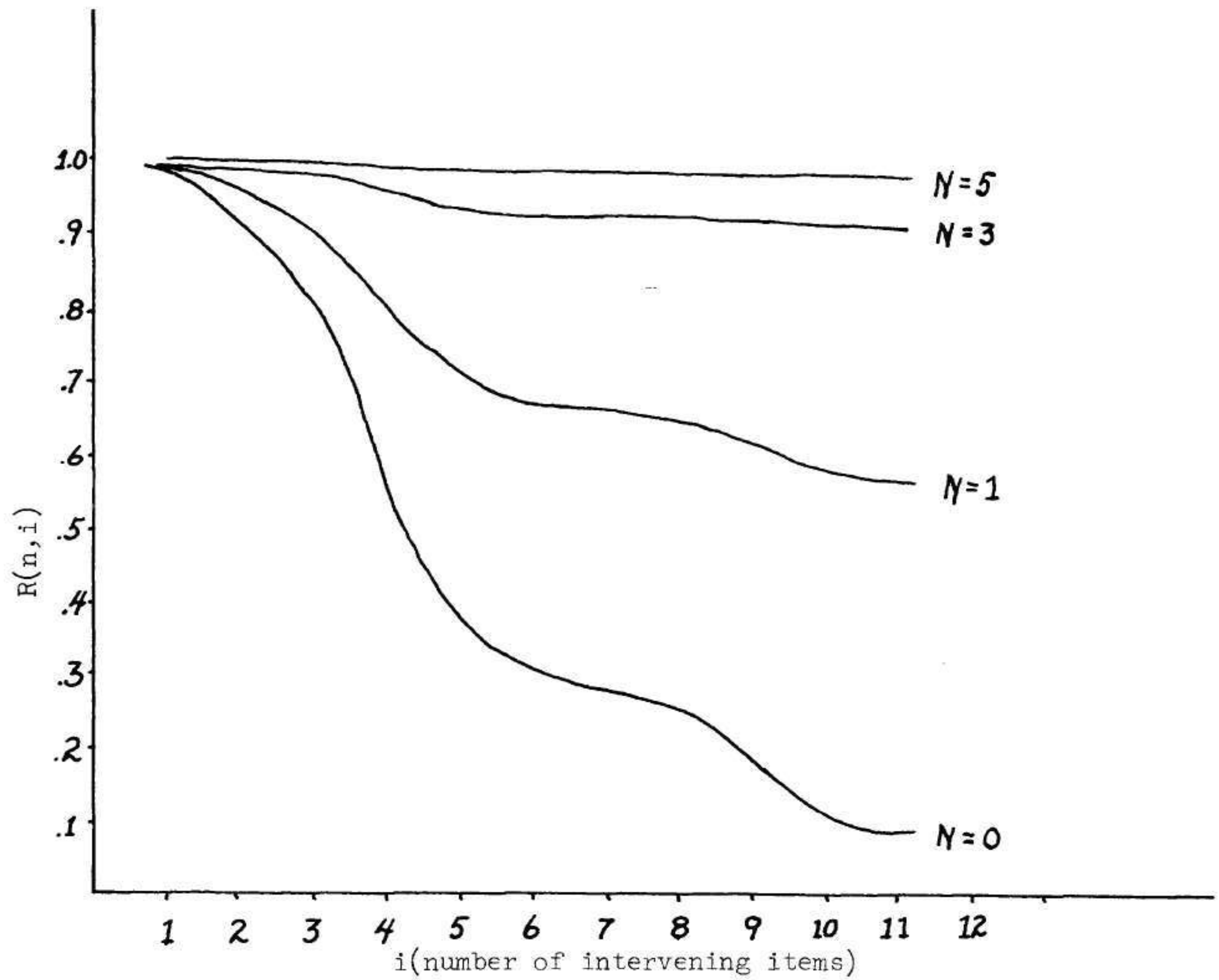


Figure 24. Family of R(n,i) Curves.

last rehearsal prior to the acquisition of the m + 1-th item. Therefore the model along with the expected number of questions analysis from Chapter I can be used to predict the number of trials S will use in a particular m item task for a particular value of V obtained from his first rehearsal. A computer program was written which for particular values of k and v would make such predictions. This required a $A(200 \times m \times 3)$ array in which was stored the trial number in the first dimension, the item number in the second, and the two counters N and i and $R^n(N,i)$ in the third dimension. Thus for every trial number there existed m triples which consisted of N,i and $R^n(N,i)$ for the m items. Until an item j was acquired the three entries $A(1,j,1) = A(1,j,2) = A(1,j,3)$ were all zero. After every trial, whether acquisition or rehearsal, all of the j-1 non-zero entries in A were tested to examine if $A(1,c,3) = R^n(N,i)$ is above threshold v for all $c \leq j - 1$. Any such entry which was at or below the threshold was put in a stack to be rehearsed on trial $\ell = \ell + 1$ or following. If several entries were indications of need for rehearsal, a decision was made as to which should be rehearsed first according to which item was lowest in probability of recall. Thus if items j and k on trial l are such that $A(1,j,3) < A(1,k,3)$, then item j is rehearsed first. In case $A(1,j,3) = A(1,k,3)$, that is $R^{nj}(n_j i_j) = R^{nk}(n_k, i_k)$, the item for which N is lowest is rehearsed first. Thus the item which has the most rapidly decreasing probability of recall is rehearsed first. This procedure of deciding the order of rehearsal in case of ties is unverified with respect to psychological validity.

The results of this program were not compared with all protocols but only with those in which S performed error-free or nearly error-free. Only protocols P-3, P-4 and P-5 of S-3; P-6 of S-3; P-7a of S-7, and P-12 of S-6 were used to compare with the output of the program. It appears that according to the least sum of squares method of fit, that the output of the model best fits the data for either k = .96 or .94 with v = .95. This is satisfying in that the predicted value of v for these protocols is .9 or above, since the first item is rehearsed before a second item is acquired in all cases. The predicted number of trials is a combination of the predicted number of acquisition trials for a binary search (which each of these protocols represents) and the predicted number of rehearsals from this program.

A micro-analysis of acquisition trials as given in Chapter III above provides not only an accounting for number of trials but also for their content. A micro-analysis of rehearsal trials ideally should also account for the content of rehearsal questions. Indeed the program described above which was used to predict the number of rehearsal questions did also provide the content of the questions. However, the output of the program with regard to question content was not sufficiently close to the content of the protocols being compared to merit inclusion. Additional refinements in the program might provide for a better fit between protocols and program output.

Table 12. Analysis of Rehearsals and Acquisitions.

| | | | PREDICTED TRIALS | | | | | |
|---|---|---|---|---|---|---|---|---|
| P | ITEMS | ACTUAL TRIALS | K = .98 | | K = .96 | | K = .94 | |
| | | | V = .96 | V = .95 | V = .96 | V = .95 | V = .96 | V = .95 |
| 3 | 5 | 13 | 14 | 14 | 14 | 14 | 14 | 14 |
| 4 | 6 | 24 | 24 | 22 | 24 | 24 | 24 | 24 |
| 5 | 7 | 24 | 29 | 30 | 33 | 29 | 34 | 29 |
| 6 | 9 | 48 | 47 | 39 | 45 | 47 | 46 | 47 |
| 7a | 12 | 81 | 70 | 66 | 72 | 72 | 73 | 72 |
| 12 | 18 | 121 | 117 | 101 | 117 | 116 | 115 | 116 |
| Sum of Squares | | | 164 | 787 | 228 | 133 | 205 | 133 |

# CHAPTER VI

## CONCLUSIONS AND PROPOSED RESEARCH

A key feature of the research reported in this dissertation is the linguistic analysis of the instructions and communication events between E and S. Pask [1973] has expressed a commitment to investigate psychological processes from a linguistic point of view. The conversational aspect of the methodology presented in this paper, along with the linguistic analysis, permits the investigation of cognitive strategies in acquiring and retaining information. The purpose of this research is to permit the collection and analysis of data in which S can display his choice behavior. This is done by providing him with a language which allows a description of his choices.

The common commitment with Pask to a linguistic analysis of S's strategies and performance is at variance in the nature of the language utilized. Pask requires a language in which S and E can both reference themselves. Pask's language also permits commands as well as a wider range of questions than the language of this methodology would permit. While it is granted that a language of such power is a desirable feature of a methodology for examining strategy and awareness, it is nevertheless contended that a simpler linguistic assumption is an appropriate starting point for investigation of the task presented in this research. It is taken to be essential that measures of information be both theoretically and practically calculable on the language chosen. This can be done for the language required in this research but it is not clear that such measures can be feasibly calculated in the larger language required for self reference and commands. It is also the case that a wider analysis of questions is somewhat difficult. Modal logics seem to be necessary to contend with questions of a broader analysis. e.g. see Cresswell [1965]. Inclusion of modal operators in the linguistic analysis would seem to make the calculation of information measures more difficult and poorly understood.

It is important for future research, however, to acknowledge that minimum linguistic assumptions carry with them extremely parsimonious theoretical analyses of S's behavior. Each linguistic assumption carries with it a certain restriction as to the kind of theoretical view that can be formulated. Noticing what these are lead to future research proposals.

An example of a theoretical limitation due to limited linguistic assumptions is a limitation imposed by the exclusion of ordering predicators in the language. Indeed the nature of <u>Subgoal</u> in Chapter III makes a tacit assumption of ordering of the stimulus items and without this assumption, the fit to the data could not be obtained. Further, it is also the case that explicit inclusion of ordering predicators in the language is necessary to insure that the information trace analysis of S's behavior does in an adequate way reflect his information state since we know from the fit of his data to a <u>Subgoal</u> analysis with tacit order assumptions that S is in fact utilizing order in his language.

Another theoretical result that is tied to language assumptions can be observed in the fact that the proof of optimality in the appendix assumes that the possible matches to a particular stimulus item are equi-probable. Therefore, if this assumption is false, the non-optimal looking behavior of some S's may in fact be optimal under other assumptions. If S is making ordering assumptions about the stimulus list and the response list and testing a hypothesis about the relatedness of the orderings, then even the linear search strategy might be optimal for these conditions.

Therefore, for the sake of future research and more adequate theoretical results the linguistic assumptions should be expanded to include ordering predicates for both stimulus and response lists. The language of the $n \times n$ experiment might be:

1. Let X and Y be two sets whose members $x_i$ and $y_i$ $(i \leq n)$ are the stimulus and response items.
2. Let $O_1$, $O_2$ and R be three binary predicates such that the domain of $O_1$ is $X \times X$, the domain of $O_2$ is $Y \times Y$ and the domain of R is $X \times Y$ (two ordering predicates and a "Match" predicate)

These predicates can then be expanded into $O_{1i}$, $O_{2i}$ and $R_i$ $(i \leq n)$ by filling the first arguments with the $x_i$ or the $y_i$ appropriately. This provides that $\mathcal{L}_n^{3n}$ shall have only unary predicates for simple information measure calculation. Such a language will allow investigation of S's hypothesis testing behavior relative to orderings on X and Y.

Work on the nature of S's hypothesis testing behavior leads to the question of concept formation displayed in S's communication. It is felt that the methodology presented in this paper may provide a procedure for distinquishing between role behavior in a learning task and concept oriented learning. Experimentation in concept behavior might include some of the following:

1. An attempt might be made to show that it is consistent to define role behavior as learning behavior that can be predicted with micro level analyses such as those of Chapter III. Coulter in a private communication with the author (Coulter [1974] indicates that an extended version of the acquisition model of Chapter III has been prepared which places acquisition in the framework of exhaustive search procedures after Sternberg [1969]. His hypothesis is that any behavior which cannot be accounted for in such an exhaustive search analysis may be concept behavior.
2. An investigation might ensue which attempts to (a) identify concept behavior in subject protocols and (b) identify the subject hypothesis being tested as well as (c) the method S uses to confirm or disconfirm the hypothesis.
3. Additional research might investigate the parameters of the task that induce concept behavior. Factors to be considered might be (a) differential effects of instructions, (b) relative amounts of information in the instructions and in the language of the experiment, (c) rule size, (d) rule parameters (i.e., one-one, one-two, etc.)
4. An investigation might be made of the effect of attempts to control concept behavior in real time applications by

changes in the above parameters. Without S's knowledge, the rule to be learned might not be fixed prior to the experiment but might be fixed by varying responses to S's questions in an attempt to control his confirmation or disconfirmation of hypothesis.

Work on the generality of the models of retention and acquisition might also be profitably pursued. This might include:

1. Extensions or refinements on these models. It has been indicated above that the model of rehearsal did not adequately predict question content. Certain refinements might lead to a better fit to protocols in this area.
2. An attempt should be made to discover to what extent a model of rehearsal and retention fitted to an S in one task generalizes to that S's behavior in other tasks and at other times. Will a model of a subject which accounts for his protocol on a particular occasion predict his behavior in another experimental session?
3. Additional work should be done with the models of acquisition and rehearsal to determine if they can be usefully employed to control S's behavior in learning tasks. It would be interesting to investigate if a model fitted to a subject can be used to present information to him in other tasks producing performance approaching error-free. Success in such an undertaking would hold promise for application to computer aided or computer managed instruction.

Several of these areas of future research and others are being examined by Coulter [1974]. In particular he is interested in the effects of external memories provided to S in the form of display items. This work might lead to refinements in the models (incorporating external memory) as well as to refinements to understanding of S's hypothesis behavior relative to display items and ordering effects.

# APPENDIX A

## SOME SAMPLE PROTOCOLS

The protocols in this appendix are included to demonstrate the nature of the data collected from participating S's. Two are included: one represents a five item task and the other a 12 item task. Both represent performances that are error free or nearly error free. Included for each i is a record of the communication event or trial along with its E-message. Also included is the increment of information to S that results and a sum of such increments.

The trials are encoded to simplify recording. The dash ("—") is interpreted to mean "matches" or "goes to"; the comma (",") is interpreted as "or" and the dot ("·") is interpreted "and". Hence "A-6" encodes a question "does A go to 6?" and "B-16,17,18" encodes a question "does B go to 16,17 or 18" or possibly "does B go to 16 or does B go to 17 or does B go to 18?". The symbols "A-6·B-15" encodes the question "does A match 6 and B match 15?"

Table 13. Sample Protocol.

P-3 S-3 5x5

| $i$ | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
|---|---|---|---|---|
| 1 | a-10,11,12 | y | .7370 | .7370 |
| 2 | a-10,11 | n | 1.5850 | 2.3220 |
| 3 | a-12 | y | 0 | 2.3220 |
| 4 | b-10,11 | n | 1.0 | 3.3220 |
| 5 | a-12 | y | 0 | 3.3220 |
| 6 | b-13 | n | 1.0 | 4.3220 |
| 7 | a-12·b-14 | y | 0 | 4.3220 |
| 8 | a-12·b-14 | y | 0 | 4.3220 |
| 9 | c-10,11 | y | .5850 | 4.9070 |
| 10 | c-10 | y | 1.0 | 5.9020 |
| 11 | a-12·b-14·c-10 | y | 0 | 5.9070 |
| 12 | d-11 | n | 1.0 | 6.9070 |
| 13 | a-12·b-14·c-10·e-11·d-13 | y | 0 | 6.9070 |

Table 14. Sample Protocol.

P-5 S-3 7x7

| $i$ | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
|---|---|---|---|---|
| 1. | A-20,21,22,23 | y | .8074 | 37.508 |
| 2. | A-20,21 | y | 1.0 | 38.508 |
| 3. | A-20 | n | 1.0 | 39.508 |
| 4. | A-21 | y | 0 | |
| 5. | B-23,24,25,26 | y | .5850 | 40.093 |
| 6. | A-21 | y | 0 | |
| 7. | B-23,24 | y | 1.0 | 41.093 |
| 8. | B-24 | n | 1.0 | 42.093 |
| 9. | A-21·B-24 | y | 0 | |
| 10. | C-23,25,26 | y | .7370 | 42.830 |
| 11. | C-25,26 | n | 1.5850 | 44.415 |
| 12. | A-21·B-24·C-23 | y | 0 | |
| 13. | A-21·B-24·C-23 | y | 0 | |
| 14. | D-25,26 | n | 1.0 | 45.415 |
| 15. | D-20 | y | 1.0 | 46.415 |
| 16. | A-21·B-24·C-23·D-20 | y | 0 | |
| 17. | A-21·B-24·C-23·D-20 | y | 0 | |
| 18. | E-25,26 | n | 1.5850 | 48.000 |
| 19. | E-22 | y | 0 | |
| 20. | A-21·B-24·C-23·D-20·E-22 | y | 0 | |
| 21. | F-25 | n | 1.0 | 49.000 |
| 22. | F-26 | y | 0 | |
| 23. | A-21·B-24·C-23·D-20·E-22·F-25·G-26 | n | 0 | |
| 24. | A-21·B-24·C-23·D-20·E-22·F-26·G-25 | y | 0 | |

Table 15. Sample Protocol.

P-10 S-2 18x18

| i | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
|---|---|---|---|---|
| 1. | 1-a,b,c | n | .2630 | 271.7550 |
| 2. | 1-d,e,f | y | 2.3219 | 274.0769 |
| 3. | 1-d | n | .5850 | 274.6619 |
| 4. | 1-e | y | 1.0 | 275.6619 |
| 5. | 1-e | y | 0 | |
| 6. | 2-a,b,c | n | .2801 | 275.9420 |
| 7. | 2-e,f | n | .1069 | 276.0489 |
| 8. | 2-g,h,i | n | .3785 | 276.4274 |
| 9. | 2-j,k,l | y | 1.7370 | 278.1644 |
| 10. | 2-j | n | .5850 | 278.7494 |
| 11. | 2-k | n | 1.0 | 279.7494 |
| 12. | 2-l | y | 0 | |
| 13. | 1-d·2-L | n | 0 | |
| 14. | 1-e 2-L | y | 0 | |
| 15. | 3-a,b,c | n | .2996 | 280.0490 |
| 16. | 3-d,f | n | .2410 | 280.2900 |
| 17. | 3-g,h,i | n | .4594 | 280.7494 |
| 18. | 3-k,m | n | .4150 | 281.1644 |
| 19. | 3-n,o,p | n | 1.00 | 282.1644 |
| 20. | 3-q | n | .5850 | 282.7494 |
| 21. | 3-4 | n | 1.0 | 283.7494 |
| 22. | 3-a,b,c | n | 0 | |
| 23. | 3-d,f | n | 0 | |
| 24. | 1-e | y | 0 | |
| 25. | 3-g,h,i | n | 0 | |
| 26. | 3-j,k | y | 0 | |
| 27. | 3-k | n | 0 | |
| 28. | 3-j | y | 0 | |
| 29. | 2-L | y | 0 | |
| 30. | 1-e | y | 0 | |
| 31. | 1-e.2-L·3-j | y | 0 | |
| 32. | 1-3·2-L·3-j | y | 0 | |
| 33. | 4-a,b,c | n | .3219 | 284.0713 |
| 34. | 4-d,f | n | .2630 | 284.3343 |
| 35. | 4-g,h,i | n | .5146 | 284.8489 |
| 36. | 4-k | n | .2224 | 285.0713 |
| 37. | 4-m,n,o | n | 1.0 | 286.0713 |

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Table 15. Sample Protocol. (continued)

P-10 S-2 18x18

| i | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
|---|---|---|---|---|
| 38. | 4-p,q,r | y | 0 | |
| 39. | 4-p | n | .5850 | 286.6563 |
| 40. | 4-q | y | 1.0 | |
| 41. | 1-e·2-L·3-j·4-r | n | 0 | |
| 42. | 1-e | y | 0 | |
| 43. | 2-L | y | 0 | |
| 44. | 3-j | y | 0 | |
| 45. | 4-q | y | 0 | |
| 46. | 5-a,b,c,d,e,f,g,h,i | y | .8074 | 288.4637 |
| 47. | 5-a,b,c, | n | .6781 | 289.1418 |
| 48. | 5-d,f | n | .7370 | 289.8788 |
| 49. | 5-g | n | .5850 | 290.4638 |
| 50. | 5-h | n | 1.0 | 291.4638 |
| 51. | 5-i | y | 0 | |
| 52. | 4-q | y | 0 | |
| 53. | 6-a,b,c,d,e,f,g,h,i | y | .8931 | 292.3569 |
| 54. | 6-a,b,c | y | 1.2224 | 293.5193 |
| 55. | 6-a | y | 1.5850 | 295.1643 |
| 56. | 1-e·2-L·3-j·4-q·5-i·6-a | y | 0 | |
| 57. | 7-a,b,c,d,e,f,g,h,i | n | 1.0 | 296.1643 |
| 58. | 7-j,k,l,m,n | y | 1.0 | 297.1643 |
| 59. | 7-k | n | .5850 | 297.7493 |
| 60. | 7-m | n | 1.0 | 298.7493 |
| 61. | 7-n | y | 0 | |
| 62. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n | y | 0 | |
| 63. | 8-a,b,c,d,e,f,g,h,i | y | .8745 | 299.6238 |
| 64. | 8-b,c | y | 1.5850 | 301.2088 |
| 65. | 8-b | n | 1.0 | 302.2088 |
| 66. | 8-c | y | 0 | |
| 67. | 1-e·2-L·3-j·4-q·5-i·7-n 8-c·6-a | y | 0 | |
| 68. | 9-a,b,c,d,e,f,g,h,i | y | 1.0 | 303.2088 |
| 69. | 9-b | n | .3219 | 303.5307 |
| 70. | 9-d | n | .4150 | 303.9457 |
| 71. | 9-e | n | 0 | |
| 72. | 9-f | y | 1.5850 | 305.5307 |
| 73. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n· 8-c·9-f | y | 0 | |

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Table 15. Sample Protocol. (continued)

| | P-10 S-2 18x18 | | | |
|---|---|---|---|---|
| i | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
| 74. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f | y | 0 | |
| 75. | 10-a,b,c,d,e,f,g,h,i | n | .8480 | 306.3787 |
| 76. | 10-j,k,l,m,n | n | .7370 | 307.1157 |
| 77. | 10-0 | y | 1.5850 | 308.7007 |
| 78. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0 | y | 0 | |
| 79. | 11-a,b,c,d,e,f,g,h,i | n | 1.0 | 309.7007 |
| 80. | 11-j,k,l,m,n | y | 1.0 | 310.7007 |
| 81. | 11-k | y | 1.0 | 311.7007 |
| 82. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k | y | 0 | |
| 83. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k | y | 0 | |
| 84. | 12-a,b,c,d,e,f,g,h,i | y | .8074 | 312.5081 |
| 85. | 12-e | n | 0 | |
| 86. | 12-d | y | 2.0 | 314.5081 |
| 87. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k·12-d | y | 0 | |
| 88. | 1-e·2-L·3-j·4-q·5-i·6-q·7-n·8-c·9-f·10-0·11-k·12-d | y | 0 | |
| 89. | 13-a,b,c,d,e,f,g,h,i | n | 1.0 | 315.5081 |
| 90. | 13-j,k,l,m,n, | y | 1.5850 | 317.0931 |
| 91. | 13-L | n | 0 | |
| 92. | 13-m | y | 0 | |
| 93. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k·12-d·13-m | y | 0 | |
| 94. | 14-a,b,c,d,e,f,g,h,i | n | 1.3219 | 318.4150 |
| 95. | 14-j,k,l,m,n | n | 0 | |
| 96. | 14-p | n | 1.0 | 319.4150 |
| 97. | 14-r | y | 0 | |
| 98. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k·12-d·13-m·14-r | y | 0 | |
| 99. | 14-r | y | 0 | |
| 100. | 13-b | n | 0 | |
| 101. | 13-d | n | 0 | |
| 102. | 13-a,b,c,d,e,f,g,h,i | n | 0 | |
| 103. | 13-j,k,l,m,n | y | 0 | |

-continued-

Table 15. Sample Protocol. (continued)

| | P-10 S-2 18x18 | | | |
|---|---|---|---|---|
| i | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
| 104. | 13-m | y | 0 | |
| 105. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k·12-d·13-m·14-r | y | 0 | |
| 106. | 1-e·2-L·3-j·4-q·5-i·6-q·7-n·8-c·9-f·10-0·11-k·12-d·13-m·14-r | y | 0 | |
| 107. | 15-a,b,c,d,e,f,g,h,i | y | .4150 | 319.8300 |
| 108. | 15-b | y | 1.5850 | 321.4150 |
| 109. | 13-m.14-5.15-b | y | 0 | |
| 110. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k·12-d·13-m·14-r·15-b | y | 0 | |
| 111. | 16-a,b,c,d,e,f,g,h,i | n | 1.5850 | 323.0 |
| 112. | 16-k,j,l,m,n | n | 0 | |
| 113. | 16-p | y | 0 | |
| 114. | 13-m.14-r.15-b.16-p | y | 0 | |
| 115. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k·12-d·13-m·14-r.15-b·16-p | y | 0 | |
| 116. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k·12-d·13-m·14-r·15-b·16-p | y | 0 | |
| 117. | 17-a,b,c,d,e,f,g,h,i | y | 0 | |
| 118. | 17-h | n | 1.0 | 324.0 |
| 119. | 17-g | y | 0 | |
| 120. | 18-h | y | 0 | |
| 121. | 16-p·17-h·18-g | n | 0 | |
| 122. | 16-p·17-g.18-h | y | 0 | |
| 123. | 13-m·14-5·15-b·16-p·17-g·18-h | y | 0 | |
| 124. | 1-e·2-L·3-j·4-q·5-i·6-q·7-n·8-c·9-f·10-0·11-k·12-d·13-m·14-r·15-b·16-p·17-g·18-h | y | 0 | |
| 125. | 1-e·2-L·3-j·4-q·5-i·6-a·7-n·8-c·9-f·10-0·11-k·12-d·13-m·14-r·15-b·16-p·17-g·18-h | y | 0 | |

Table 16. Sample Protocol.

P-12 S-3 18x18

| $i$ | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
|---|---|---|---|---|
| 1. | A-10,11,12,13,14,15,16,17,18 | n | 1.0 | 272.492 |
| 2. | A-5,6,7,8,9 | y | .8480 | 273.340 |
| 3. | A-7,8,9 | n | 1.3219 | 274.662 |
| 4. | A-6 | y | 1.0 | 275.662 |
| 5. | A-6 | y | 0 | |
| 6. | B-10,11,12,13,14,15,16,17,18 | y | .9175 | 276.579 |
| 7. | B-14,15,16,17,18 | y | .8480 | 277.427 |
| 8. | B-16,17,18 | n | 1.3219 | 278.749 |
| 9. | B-14 | n | 1.0 | 279.749 |
| 10. | B-15 | y | 0 | |
| 11. | B-15·A-6 | y | 0 | |
| 12. | B-15·A-6 | y | 0 | |
| 13. | C-10,11,12,13,14,15,16,17,18 | n | 1.0 | 280.749 |
| 14. | C-5,6,7,8,9 | y | 1.0 | 281.749 |
| 15. | C-5 | n | .4150 | 282.164 |
| 16. | C-8,9 | y | .5850 | 282.749 |
| 17. | C-9 | n | 1.0 | 283.749 |
| 18. | C-8 | y | 0 | |
| 19. | A-6·B-15·C-8 | y | 0 | |
| 20. | A-6·B-15·C-8 | y | 0 | |
| 21. | D-10,11,12,13,14,15,16,17,18 | y | .9069 | 284.656 |
| 22. | D-14,15,16,17,18 | n | 1.0 | 285.656 |
| 23. | D-12,13 | y | 1.0 | 286.656 |
| 24. | D-13 | n | 1.0 | 287.656 |
| 25. | D-12 | y | 0 | |
| 26. | A-6·B-15·C-8·D-12 | y | 0 | |
| 27. | A-6·B-15·C-8·D-12 | y | 0 | |
| 28. | E-10,11,12,13,14,15,16,17,18 | n | 1 | 288.656 |
| 29. | E-5,6,7,8,9 | n | .8074 | 289.464 |
| 30. | E-3,4 | n | 1.00 | 290.464 |
| 31. | E-2 | n | 1.00 | 291.464 |
| 32. | E-1 | y | 0 | |
| 33. | E-1·A-6·B-15·C-8·D-12 | y | 0 | |
| 34. | F-10,11,12,13,14,15,16,17,18 | n | 1.1155 | 292.579 |
| 35. | F-5,6,7,8,9 | y | 1.0 | 293.579 |
| 36. | F-5,7 | n | 1.5850 | 295.164 |
| 37. | F-9 | y | 0 | |
| 38. | A-6·B-15·C-8·D-12·E-1·F-9 | y | 0 | |
| 39. | F-9·E-1 | y | 0 | |

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Table 16. Sample Protocol. (Continued)

P-12 S-3 18x18

| i | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
|---|---|---|---|---|
| 40. | G-12,13,14,15,16,17,18 | y | 1.2630 | 296.427 |
| 41. | G-16,17,18 | y | .7370 | 297.164 |
| 42. | G-17,18 | y | .5850 | 297.749 |
| 43. | G-18 | n | 1.0 | 298.749 |
| 44. | G-17 | y | 0 | |
| 45. | F-9 | y | 0 | |
| 46. | A-6·B-15·C-8·D-12·E-1·F-9·G-18 | n | 0 | |
| 47. | G-17 | y | 0 | |
| 48. | G-17 | y | 0 | |
| 49. | H-12,13,14,15,16,17,18 | y | 1.4594 | 300.208 |
| 50. | H-12,13,14 | n | 1.0 | 301.208 |
| 51. | H-16 | n | 1.0 | 302.208 |
| 52. | H-18 | y | 0 | |
| 53. | H-18·G-17 | y | 0 | |
| 54. | F-9·E-1 | y | 0 | |
| 55. | I-10,11,12,13,14,15,16,17,18 | n | 1.0 | 303.208 |
| 56. | I-4,5,6,7,8,9 | y | .7370 | 303.946 |
| 57. | I-4,5 | y | .5850 | 304.531 |
| 58. | I-5 | y | 1.0 | 305.531 |
| 59. | I-5 | y | 0 | |
| 60. | H-18·G-17 | y | 0 | |
| 61. | A-6·B-15·C-8·D-12·E-1·F-9·G-17·H-18·I-5 | y | 0 | |
| 62. | E-1·F-9·G-17·H-18·I-5 | y | 0 | |
| 63. | J-10,11,12,13,14,15,16,17,18 | n | 1.1699 | 306.700 |
| 64. | J-1,2,3 | y | 1.0 | 307.700 |
| 65. | J-3 | y | 1.0 | 308.700 |
| 66. | J-3 | y | 0 | |
| 67. | E-1·F-9·G-17·H-18·I-5·J-3 | y | 0 | |
| 68. | I-5·J-3 | y | 0 | |
| 69. | K-10,11,12,13,14,15,16,17,18 | y | .6781 | 309.378 |
| 70. | K-13,14,15,16,17,18 | n | 1.3219 | 310.700 |
| 71. | K-10,11 | y | 0 | |
| 72. | K-10 | n | 1.0 | 311.700 |
| 73. | K-11 | y | 0 | |
| 74. | G-17·H-18·I-5·J-3·K-11 | y | 0 | |
| 75. | G-17·H-18·I-5·J-3·K-11 | y | 0 | |

-continued-

Table 16. Sample Protocol. (Continued)

P-12 S-3 18x18

| i | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
|---|---|---|---|---|
| 76. | A-6·B-15·C-8·D-12·E-1·F-9·<br>G-17·H-18·I-5·J-3·K-11 | y | 0 | |
| 77. | D-12·E-1·F-9 | y | 0 | |
| 78. | L-10,11,12,13,14,15,16,17,18 | n | 1.2223 | 312.921 |
| 79. | L-5,6,7,8,9 | y | 1.0 | 313.507 |
| 80. | L-2 | y | 1.0 | 314.507 |
| 81. | L-2 | y | 0 | |
| 82. | G-17·H-18·I-5·J-3·K-11·L-2 | y | 0 | |
| 83. | A-6·B-15·C-8·D-12·E-1·F-9·G-17·<br>H-18·I-5·J-3·K-11·L-2 | y | 0 | |
| 84. | M-10,11,12,13,14,15,16,17,18 | y | .5850 | 315.093 |
| 85 | M-10,11,12,13,14 | y | .4150 | 315.508 |
| 86. | M-14 | n | .5850 | 316.093 |
| 87. | M-10 | n | 1.0 | 317.093 |
| 88. | M-13 | y | 0 | |
| 89. | L-2·M-13 | y | 0 | |
| 90. | M-13 | y | 0 | |
| 91. | I-5·J-3·K-11·L-2·M-13 | y | 0 | |
| 92. | N-10,11,12,13,14,15,16,17,18 | n | 1.3219 | 318.415 |
| 93. | N-7,8,9 | y | 1.0 | 3.9.415 |
| 94. | N-7 | y | 0 | |
| 95. | N-7 | y | 0 | |
| 96. | M-13 | y | 0 | |
| 97. | L-2 | y | 0 | |
| 98. | K-11 | y | 0 | |
| 99. | N-7 | y | 0 | |
| 100. | M-13 | y | 0 | |
| 101. | A-6·B-15·C-8·D-12·E-1·F-9·<br>G-17·H-18·I-5·J-3·K-11·L-2·<br>M-13·N-7 | y | 0 | |
| 102. | 0-5,10 | y | 1.0 | 320.415 |
| 103. | 0-10 | y | 1.0 | 321.415 |
| 104. | 0-10 | y | 0 | |
| 105. | N-7·M-13 | y | 0 | |
| 106. | D-12 | y | 0 | |
| 107. | 0-10·N-7·M-13 | y | 0 | |
| 108. | A-6·B-15·C-8·D-12·E-1·<br>F-9·G-17·H-18·I-5·J-3·<br>K-11·L-2·M-13·N-7·0-10 | y | 0 | |

-continued-

Table 16. Sample Protocol. (Continued)

P-12 S-3 18x18

| $i$ | $Q_i$ | Ans. | $Inc_i$ | $\sum_i Inc_i$ |
|---|---|---|---|---|
| 109. | P-4,16 | y | .5850 | 322.00 |
| 110. | P-16 | y | 1.0 | 323.00 |
| 111. | P-16 | y | 0 | |
| 112. | O-10 | y | 0 | |
| 113. | P-16.O-10.N-7·M-13 | y | 0 | |
| 114. | P-16 | y | 0 | |
| 115. | A-6.B-15·C-8·D-12·E-1·F-9·<br>G-17·H-18·I-5·J-3·K-11·L-2·<br>M-13·N-7.O-10.P-16 | y | 0 | |
| 116. | Q-4 | y | 1.0 | 324.00 |
| 117. | Q-4 | y | 0 | |
| 118. | P-16·O-10·N-7 | y | 0 | |
| 119. | A-6·B-15·C-8·D-12·E-1·<br>E-9·G-17·H-18·I-5·J-3·<br>K-11·L-2·M-13·N-7·O-10·<br>P-16·Q-4 | y | 0 | |
| 120. | A-6·B-15·C-8·D-12·E-1·<br>F-9·G-17·H-18·I-5·J-3·K-11·<br>P-16·O-10·Q-4 | y | 0 | |
| 121. | A-6·B-15·C-8·D-12·E-1·F-9·<br>G-17·H-18·I-5·J-3·K-11·L-2·<br>M-13·N-7·O-10·P-16·Q-4·R-14 | y | 0 | |

# APPENDIX B

## INSTRUCTIONS TO THE SUBJECTS

The following instructions were read to the participating Ss:

We are interested in human information processing models of learning. The goal of this experiment is to discover how efficiently individuals can learn materials without error. The experiment you are participating in is a simple learning task.

You will be given a list of numbers and a list of single letters. One-to-one associations among the letters and numbers have been previously assigned. That is, each number is associated with one letter and vice versa. There are an equal number of items in each list. Your task is to learn which number goes with which letter. You will have completed the task when you can give all pairs of letters and numbers without interruption.

For example, suppose the numbers are 10, 11, 12, and 13, and the letters are W,X,Y, and Z. Assume that W goes with 13, X goes with 11, Y goes with 12 and Z goes with 10. [show subject a paper exhibiting these pairs.]* At the beginning of the experiment only the experimenter knows which number goes with which letter. You must discover the correct pairs by asking the experimenter questions. There are three kinds of questions you can ask: AND questions, OR

*Statements in brackets [ ] are not read to the subject. They are to remind the experimenter of certain actions and/or for explanatory purposes.

questions, and negations of such questions. Of course you may also ask simple questions such as "does X go with 10?" In our example, the experimenter would answer "NO" because X goes with 11.

A subject might ask a more complicated question such as the OR question "does X go with 10 or 11?" In this case the answer is YES since X goes with 11. A question such as "does X go with 11 and Y go with 10" is an AND question. The answer is NO because Y goes with 12.

To be sure you understand how to interpret these various types of questions, please let me ask you some questions based on the example before you. Please respond YES or NO to my question.

Does Y go with 11, 12, or 13?

Does Y go with 12 or does X go with 13?

Does W go with 13 and X go with 10?

Does X go with 11 and Y go with 12?

It is very important that you try to learn these pairs without making any errors. Once you have learned a correct pair, you should not forget it before you learn all of the other pairs. This will undoubtedly mean that you must rehearse previously acquired information; that is, you may repeatedly ask about previously acquired information to insure that you have not forgotten those pairs. Consider the example concerning the numbers 10, 11, 12, and 13, and the letters W,X,Y,Z. Suppose the subject has learned that W goes with 13 and that X goes with 11. Before finding the matches for Y and Z, the subject may choose to ask the rehearsal question "does W go with 13 and X go with 11" in order to be sure he has not forgotten those pairs. A rehearsal question is usually an AND question.

Rehearse anytime you feel you must. It is not necessary to try to minimize rehearsals - rehearse as often as you want. You may rehearse as many of the previously learned pairs as you want to. Also, I would like for you to vocalize all rehearsals so that I can inform you whether or not your rehearsal is correct.

It is also an error to ask about a previously learned item while searching for some other item. For example, suppose you had already learned that X goes with 11. Then you should not ask "does 12 go with X or Y" because the match for X has already been determined.

Do you have any questions?

To be sure I have properly explained the task to you, I would like for you to learn a small number of pairs by the procedure described to you. [During the trial session, an attempt is made to determine if the subject understands the task by observing his performance. Also, this allows the subject to clarify any misunderstanding.]

[Past experimentation has indicated that it is not always clear to the experimenter when a subject has completed a rehearsal. This is because a subject may rehearse all or only portions of previously acquired information. Therefore, during the trial run, the subject will be requested to indicate the end of a rehearsal question and to continue these signals in the following experiment.]

You are to learn pairs. Your task is to match the numbers in this list [Show the list to S and read it] to the letters in this list

[Show the list to S and read it] [Allow several seconds for familiarization] Your task is to match the numbers to the letters by means of the kind of "YES"/"NO" questions we have discussed. Please remember that the rule that you are to discover is a one-one rule. This means that one number matches exactly one letter and that no two numbers match the same letter. There is for each number a unique letter.

I might add that the number-letter pairs were assigned randomly so you should not try to discover or predict a pattern or rule which will relate the pairs.

Please remember to rehearse sufficiently to complete the task with as few mistakes as possible. When you can state all of the pairs in a question which is answered "YES" you will have finished the task.

You may begin.

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# VITA

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